Quantum Random Walks

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Universidad de los Andes

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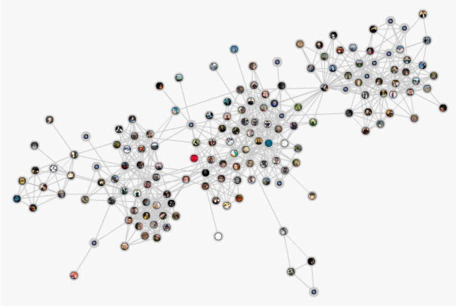

Quantum Walk Search

- Quantum walk search algorithms

A Quantum Random Walk Search Algorithm

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(Dated: February 1, 2008)

Quantum random walks on graphs have been shown to display many interesting properties, including exponentially fast hitting times when compared with their classical counterparts. However, it is still unclear how to use these novel properties to gain an algorithmic speed-up over classical algorithms. In this paper, we present a quantum search algorithm based on the quantum random walk architecture that provides such a speed-up. It will be shown that this algorithm performs an oracle search on a database of N items with $O\left(\sqrt{N}\right)$ calls to the oracle, yielding a speed-up similar to other quantum search algorithms. It appears that the quantum random walk formulation has considerable flexibility, presenting interesting opportunities for development of other, possibly novel quantum algorithms.

Quantum Computation

Universal computation by multi-particle quantum walk

Andrew M. Childs,[1,2] David Gosset,[1,2] Zak Webb[2,3]

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Abstract

A quantum walk is a time-homogeneous quantum-mechanical process on a graph defined by analogy to classical random walk. The quantum walker is a particle that moves from a given vertex to adjacent vertices in quantum superposition. Here we consider a generalization of quantum walk to systems with more than one walker. A continuous-time multi-particle quantum walk is generated by a time-independent Hamiltonian with a term corresponding to a single-particle quantum walk for each particle, along with an interaction term. Multi-particle quantum walk includes a broad class of interacting many-body systems such as the Bose-Hubbard model and systems of fermions or distinguishable particles with nearest-neighbor interactions. We show that multi-particle quantum walk is capable of universal quantum computation. Since it is also possible to efficiently simulate a multi-particle quantum walk of the type we consider using a universal quantum computer, this model exactly captures the power of quantum computation. In principle our construction could be used as an architecture for building a scalable quantum computer with no need for time-dependent control.

- Estudio de fenómenos como:

Transferencia de energía dentro de sistemas fotosintéticos

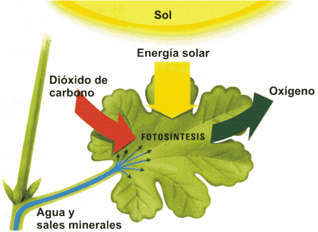

Figura: Tomado de [fot16]

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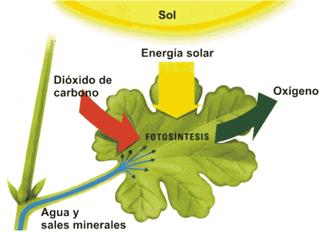

Figura: Tomado de [fot16]

Transiciones cuánticas topológicas

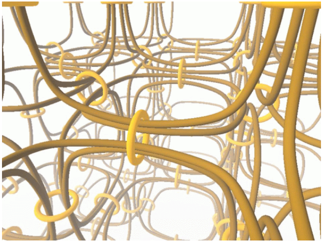

Figura: Tomado de [PS16]

- Transiciones cuánticas topológicas

Statistical moments of quantum-walk dynamics reveal topological quantum transitions

Filippo Cardano,[1] Maria Maffei,[1] Francesco Massa,[1,*] Bruno Piccirillo,[1] Corrado de Lisio,[1,2] Giulio De Filippis,[1,2] Vittorio Cataudella,[1,2] Enrico Santamato,[1] and Lorenzo Marrucci[a,1,3]

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Abstract

Go to:

Many phenomena in solid-state physics can be understood in terms of their topological properties. Recently, controlled protocols of quantum walk (QW) are proving to be effective simulators of such phenomena. Here we report the realization of a photonic QW showing both the trivial and the non-trivial topologies associated with chiral symmetry in one-dimensional (1D) periodic systems. We find that the probability distribution moments of the walker position after many steps can be used as direct indicators of the topological quantum transition: while varying a control parameter that defines the system phase, these moments exhibit a slope discontinuity at the transition point. Numerical simulations strongly support the conjecture that these features are general of 1D topological systems. Extending this approach to higher dimensions, different topological classes, and other typologies of quantum phases may offer general instruments for investigating and experimentally detecting quantum transitions in such complex systems.

Outline

1. Motivación
2. Fundamentos
   - Classical Walk
   - Quantum Walk
     - Continuous Time Quantum Walk
     - Discrete Time Quantum Walk
3. Implementación
   - Quincunx-Linear Optics
   - Quantum Optics

Classical Walk

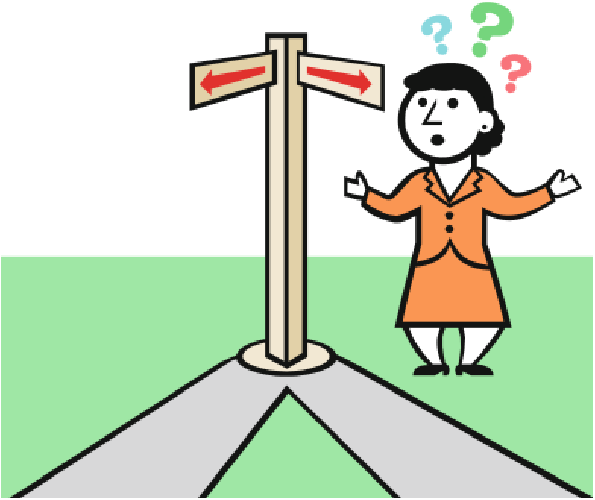
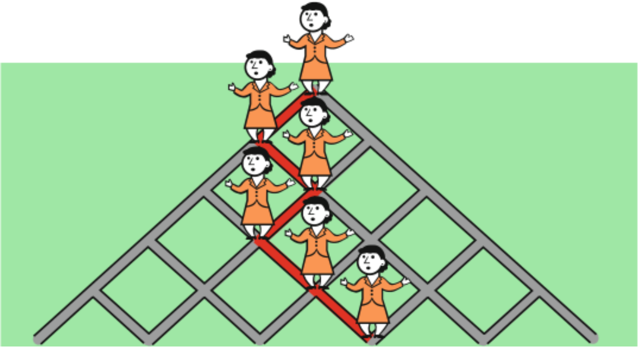


Figura: Tomado de [MW13]

Classical Walk



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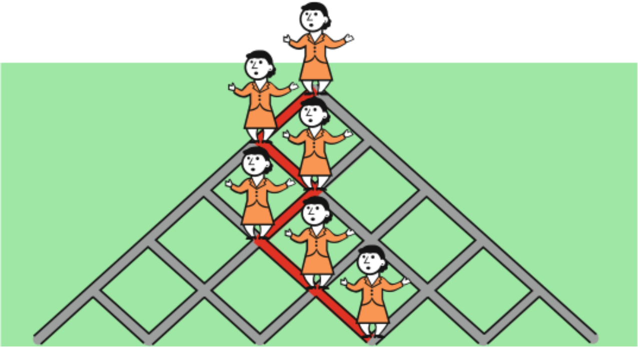
Classical Walk



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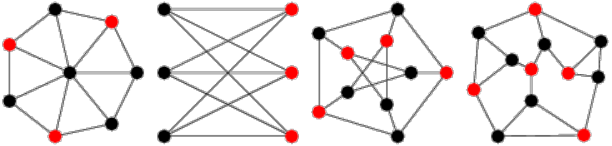
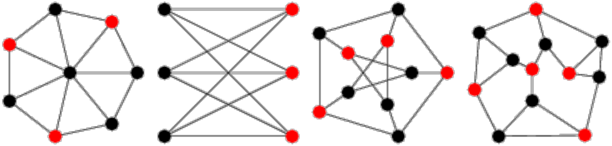


Las entradas M_{ij} de M indican la probabilidad de transición del vértice i al vértice j



Siendo $\vec{p}^t = (p_1^t, p_2^t, ..., p_{|V|}^t)$ la distribución de probabilidad para los vértices en V para un tiempo $t \rightarrow$

$$p_i^{t+1} = \sum_j M_{ij} p_j^t$$

$$\vec{p}^{t+1} = M \vec{p}^t$$

Classical Walk

- Para un círculo de N vertices

$$M = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 1 \\ 1 & 0 & 1 & 0 & \dots & 0 \\ & & \dots & & & \\ 0 & \dots & 1 & 0 & 1 & 0 \\ 0 & \dots & 0 & 1 & 0 & 1 \\ 1 & 0 & \dots & 0 & 1 & 0 \end{pmatrix}$$

Las actualizaciones solo ocurren en tiempos discretos.

Classical Walk

- Para marcha continua en tiempo
  - Transiciones en todo tiempo.
  - $\gamma \equiv$ "Jumping rate"de un vertice a su vecino.
  - γ constante independiente del tiempo.

Classical Walk

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  - **Transiciones entre nodos vecinos ocurren con probabilidad γ por unidad de tiempo**.

Classical Walk

- Matriz infinitesimal generadora del proceso:

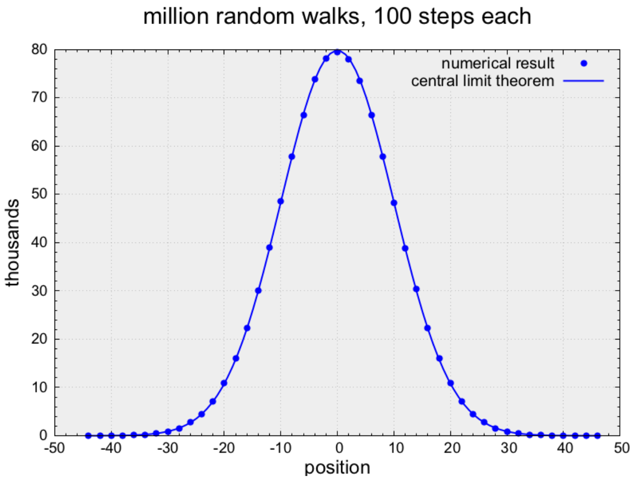
$$H_{ij} = \begin{cases} -\gamma & i \neq j \ \wedge \ \exists E(i,j) \\ 0 & i \neq j \ \wedge \ \nexists E(i,j) \\ 0 & i = j \end{cases}$$

H juega un papel similar a M.

Classical Walk

$$\frac{dp_i(t)}{dt} = -\sum_j H_{ij} p_j(t)$$

$$\frac{d\vec{p}(t)}{dt} = -H\vec{p}(t)$$

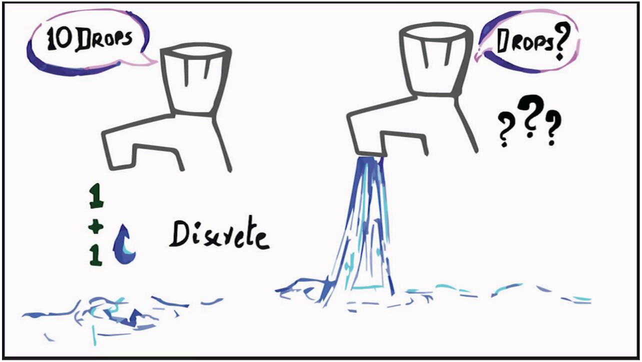


Para N pasos de la marcha aleatoria clásica :

$$\sigma = O(\sqrt{N})$$

Outline

- Continuous time quantum walk
- Discrete time quantum walk



Continuous Time Quantum Walk

- Marcha se define en un espacio de Hilbert de posición $\mathcal{H}_p$

Continuous Time Quantum Walk

- Matriz infinitesimal generadora del proceso de la marcha continua clásica se toma como el **Hamiltoniano** del proceso cuántico:

$$U(t) = e^{-\frac{i}{\hbar}\hat{H}t}$$

Quantum Computation and Decision Trees

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Discrete Time Quantum Walk

Coins Make Quantum Walks Faster

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February 1, 2008

Abstract

We show how to search N items arranged on a $\sqrt{N} \times \sqrt{N}$ grid in time $O(\sqrt{N} \log N)$, using a discrete time quantum walk. This result for the first time exhibits a significant difference between discrete time and continuous time walks without coin degrees of freedom, since it has

Discrete Time Quantum Walk

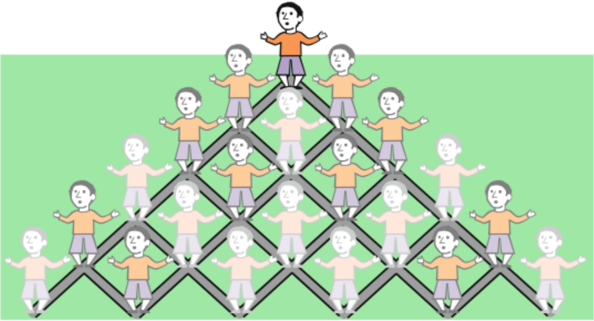

Figura: Tomado de [MW13]

Discrete Time Quantum Walk - 1D

- Espacio unidimensional sobre el cual se realiza la caminata aleatoria es una línea.

Discrete Time Quantum Walk - 1D

- Dos sistemas cuánticos:
  - Caminante.(Partícula)
  - Moneda

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Caminante

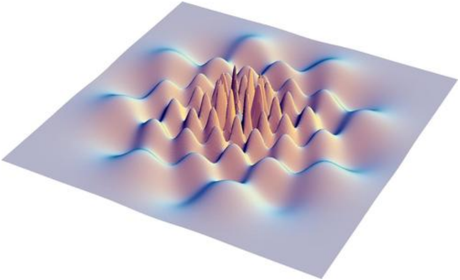
- 'Vive' en un espacio de Hilbert $\mathcal{H}_p$ con dimensión posiblemente infinita pero contable.
    - Común que base canónica de $\mathcal{H}_p$, $\{|i\rangle_p : i \in \mathbb{Z}\}$ sea las posiciones del sistema cuántico

Moneda

- El espacio de Hilbert correspondiente a la moneda $\mathcal{H}_c$ se toma como generado por los dos estados base : $\{|0\rangle_c, |1\rangle_c\}$

Quantum Walk

- Estados completos del sistema se encuentran en el espacio de Hilbert $\mathcal{H} = \mathcal{H}_c \otimes \mathcal{H}_p$.



Evolución Temporal

- Aplicación del operador unitario de moneda al estado moneda seguido de la aplicación del operador condicional de movimiento al estado completo

$$\hat{U} = \hat{S}\left(\hat{C} \otimes \mathbb{I}_p\right)$$

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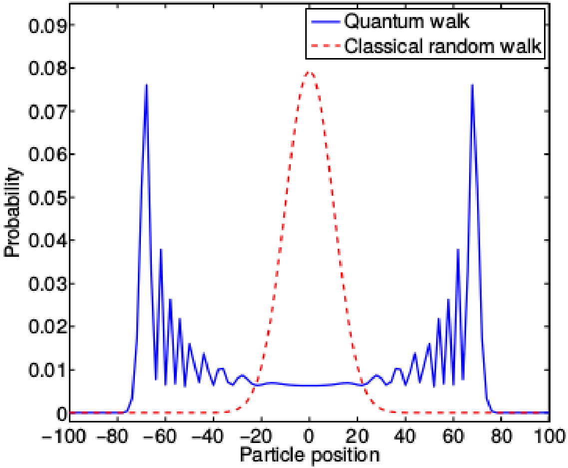

Figura: Tomado de [Cha10]

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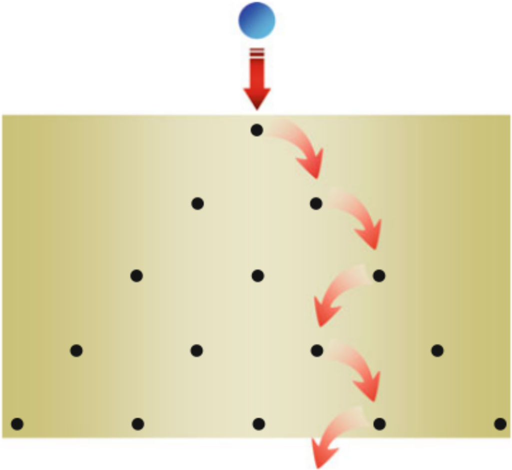
Quincunx-Linear Optics



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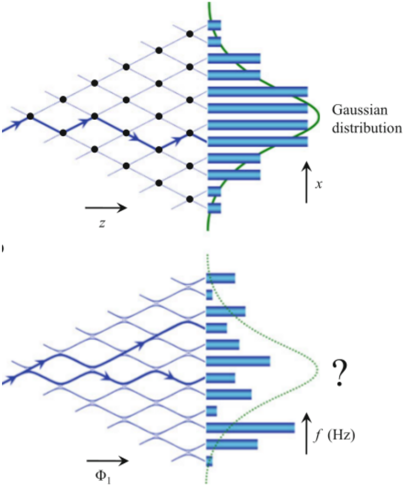
Quincunx-Linear Optics



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Quincunx-Linear Optics

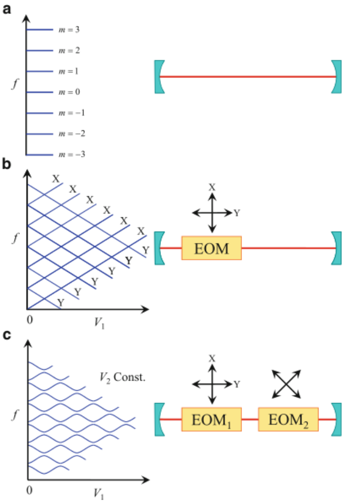
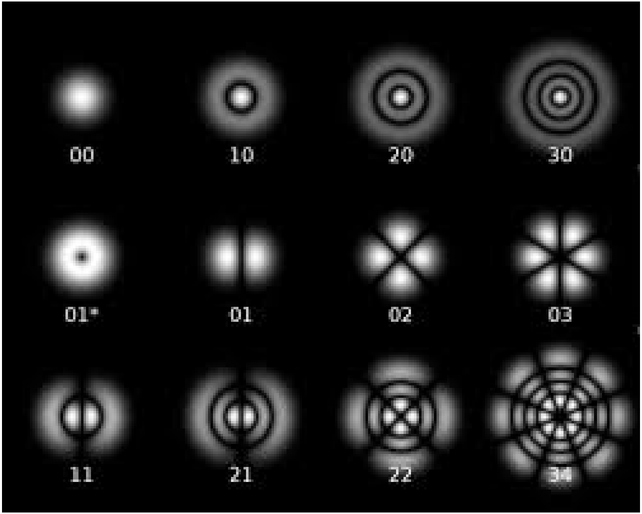


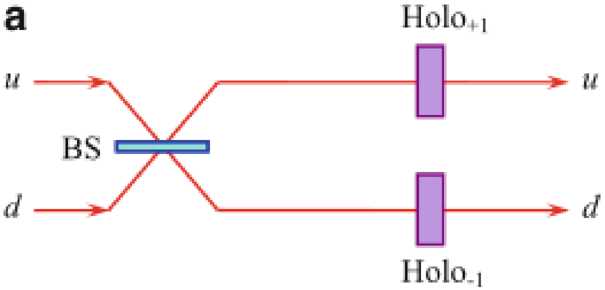
Figura: Tomado de [MW13]

Outline

Quantum Optics



Quantum Optics



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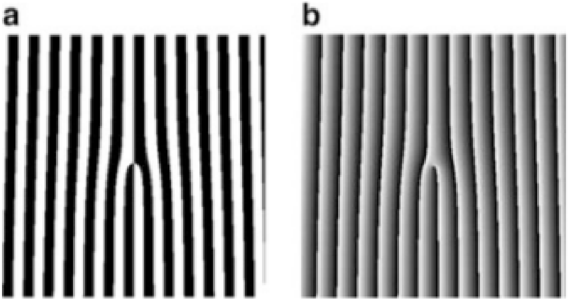
Quantum Optics



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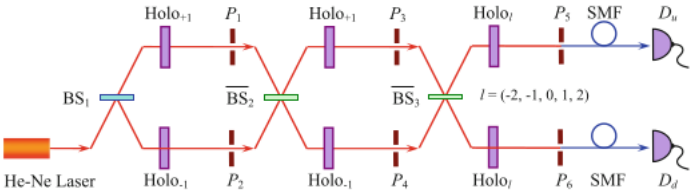
Quantum Optics



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