Universidad de los Andes
Colombia

Performing sequential WMs to reconstruct the density matrix of a two-level system

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Outline

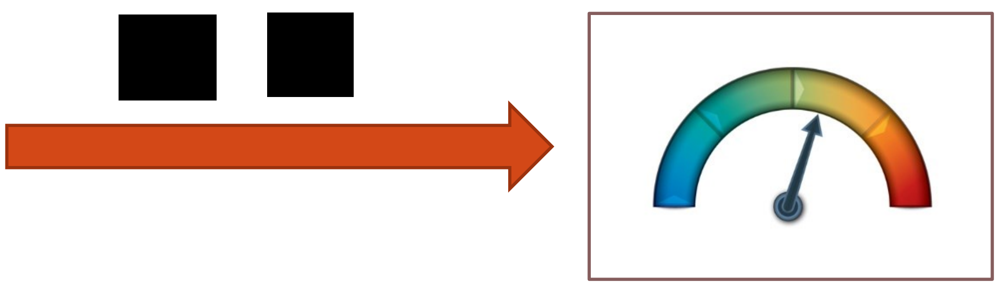
- Initial considerations – Elements of reality
- Quantum measurements: Strong vs weak
- Standard quantum state tomography vs Direct Measurement
- In brief: More questions than conclusions

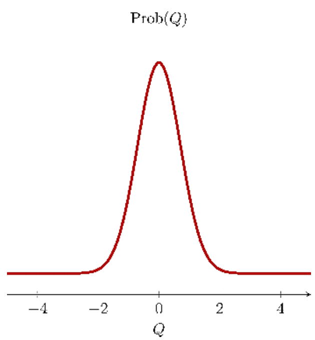
Elements of reality

Vaidman's *elements of reality*

"For any definite result of a measurement there is corresponding element of reality."

As *definite shift of the probability distribution of the pointer variable*.





[L. Vaidman. *Foundations of Physics. Vol. 26. No. 7. 1996*]

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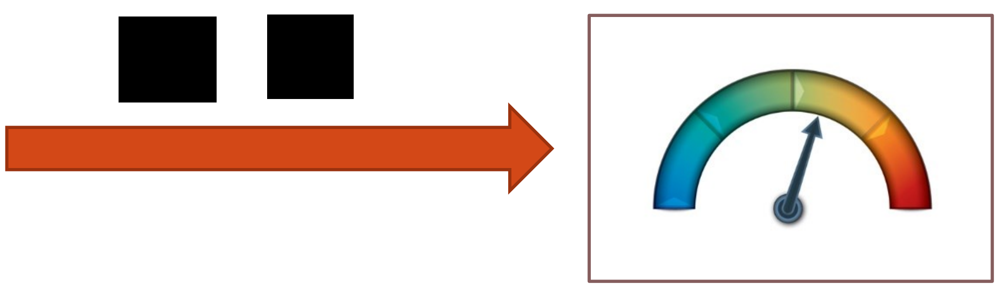

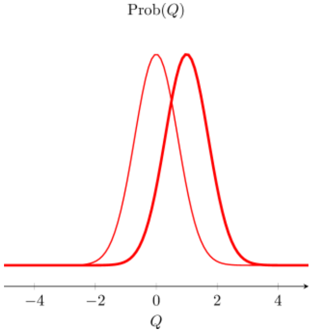

[L. Vaidman. *Foundations of Physics. Vol. 26. No. 7. 1996*]

QUANTUM MEASUREMENT

[Von Neumann approach]

Consists on

Coupling two systems

$$\hat{H}_{\text{int}} = g(t)\hat{A} \otimes \hat{P}_d$$

$$g(t) \sim g_0 \delta(t)$$

[Naimark extension]

$$\text{prob}(Q)_{\text{after}} = \frac{e^{-(Q-a_i)^2/\Delta^2}}{(\Delta^2 \pi)^{1/2}}$$

Measuring device (Meter)

e.g. Spatial or temporal profile of a beam

$$\text{prob}(Q)_{\text{in}} = \frac{e^{-Q^2/\Delta^2}}{(\Delta^2 \pi)^{1/2}}$$

Measured system

e.g. Polarization of light

The reading of the pointer variable in the end of the measurement almost always yields the value of the shift (the eigenvalue of the observable)

QUANTUM MEASUREMENT

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Measured system

e.g. Polarization of light

[Naimark extension]

Decreasing strength coupling

Or increasing the spread in the pointer distribution

$$\Delta \gg a_i$$

WEAK MEASUREMENT

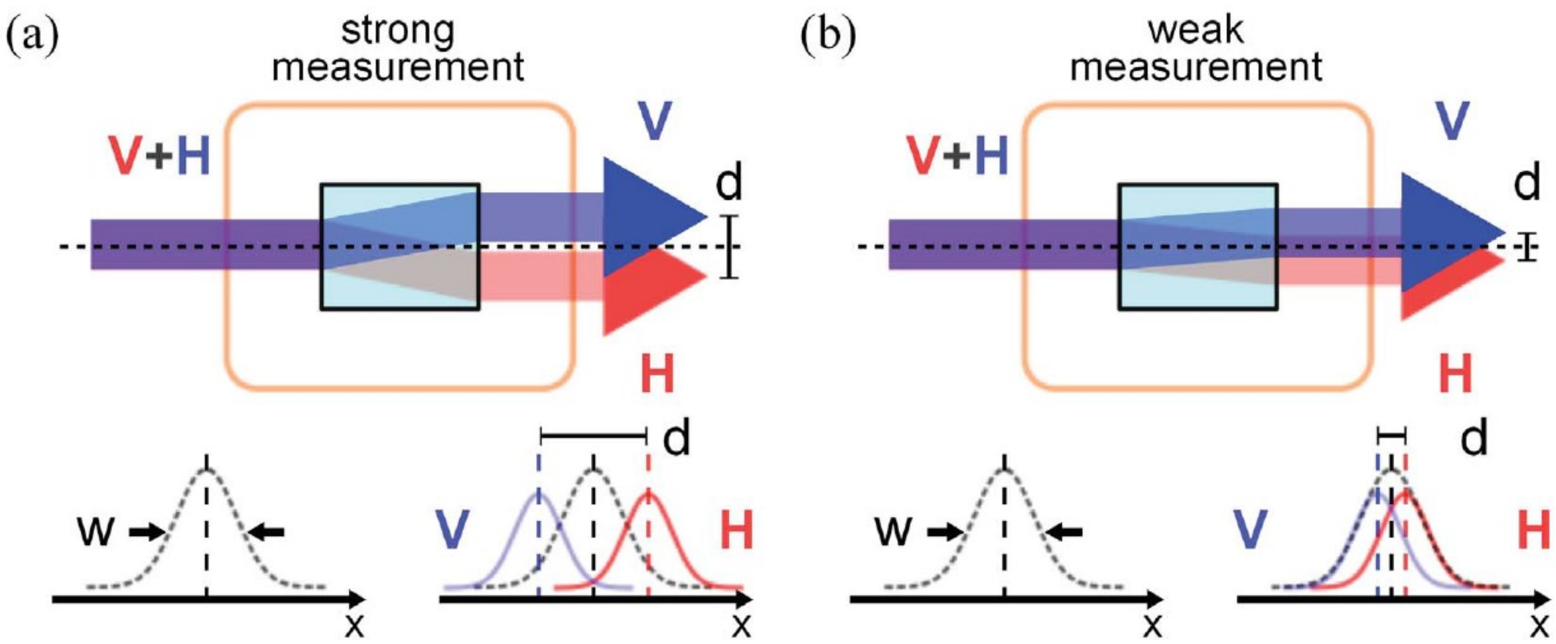
$$\text{prob}(Q) \approx \frac{e^{-(Q-A_{\text{w}})^2/\Delta^2}}{(\Delta^2\pi)^{1/2}}$$

$$A_{\text{w}} = \sum |\alpha_i|^2 a_i = \langle\Psi| A |\Psi\rangle$$

Introduces a new concept called *Weak Value*, a complex number with different interpretations and applications.

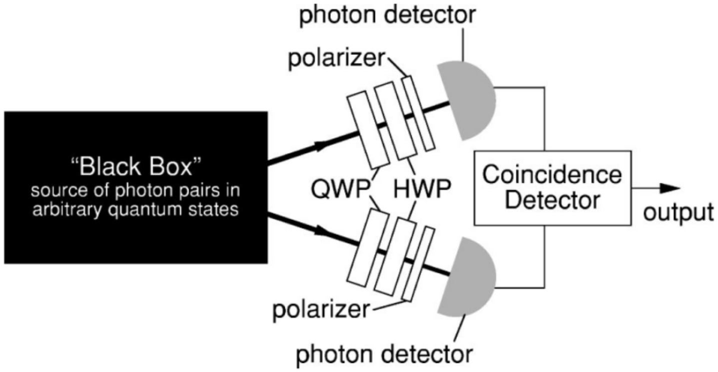
weak-measurement element of reality

Strong vs weak Measurements



[L. J. Salazar-Serrano. Opt. Express **23**, 10097 (2015)]

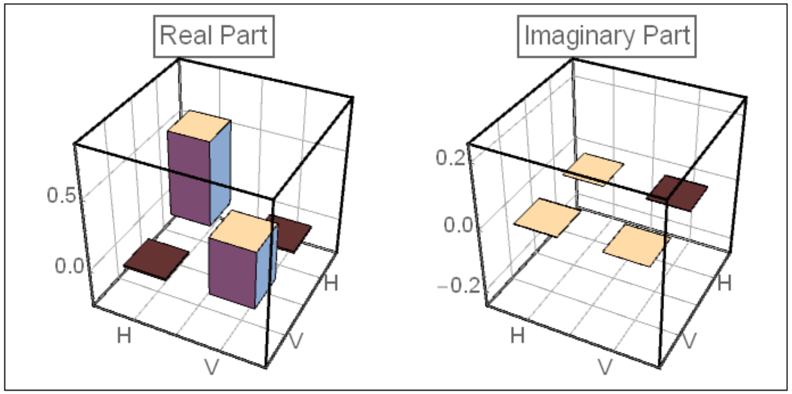
Standard Quantum State Tomography

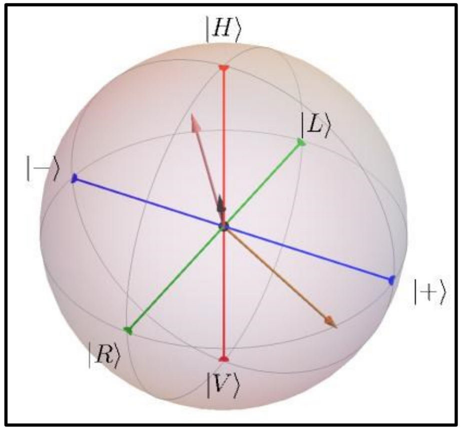


Single Qubit SQST

Projection states	***QWP***	***HWP***
H	0	0
R	0	22.5
L	0	-22.5
A	45	-22.5

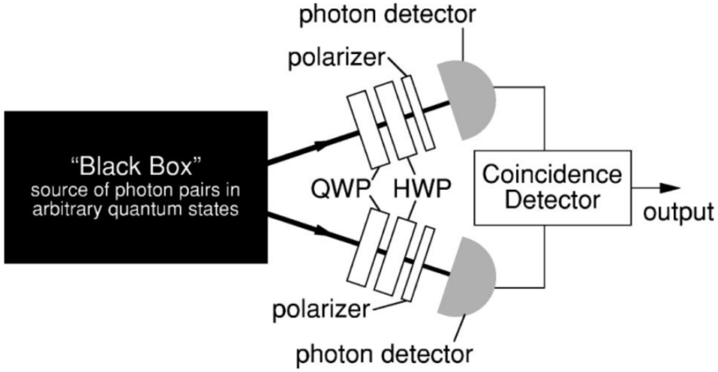
Reconstruted Polarization State – Intersection part SPDC rings

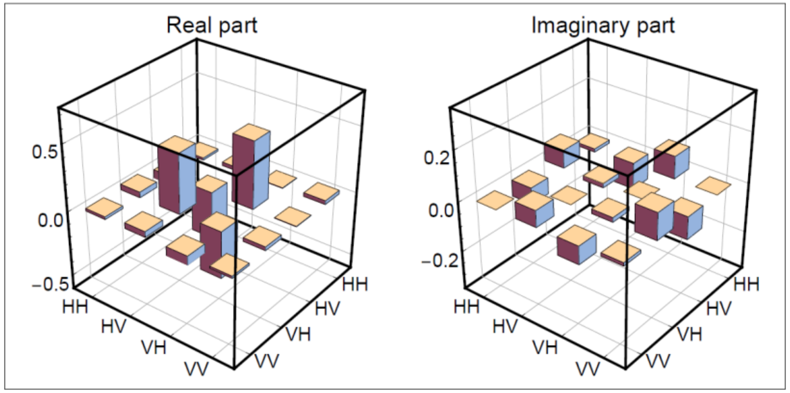




[D. F. V. James et al. Phys. Rev. A, **64**, 052312 (2011)]

Standard Quantum State Tomography





Maximum Likelihood!

Two Qubit SQST

Projections	*QWPA*	*HWPA*	*QWPB*	*HWPB*
HH	0	0	0	0
HV	0	0	0	45
HD	0	0	45	22.5
HR	0	0	0	22.5
VH	0	45	0	0
VV	0	45	0	45
VD	0	45	45	22.5
VR	0	45	0	22.5
DH	45	22.5	0	0
DV	45	22.5	0	45
DD	45	22.5	45	22.5
DR	45	22.5	0	22.5
RH	0	22.5	0	0
RV	0	22.5	0	45
RD	0	22.5	45	22.5
RR	0	22.5	0	22.5

[D. F. V. James et al. Phys. Rev. A, **64**, 052312 (2011)]

Direct measurement of the density matrix

Selected for a Viewpoint in *Physics*
PRL **117,** 120401 (2016) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2016

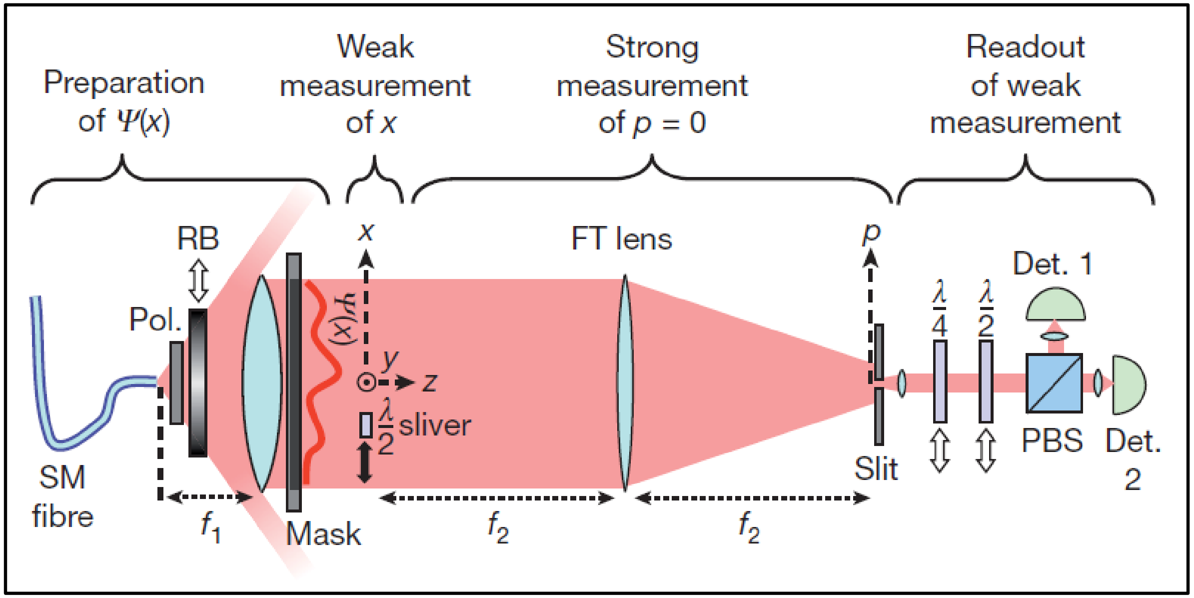
Direct Measurement of the Density Matrix of a Quantum System

G. S. Thekkadath, L. Giner, Y. Chalich, M. J. Horton, J. Banker, and J. S. Lundeen
Department of Physics and Max Planck Centre for Extreme and Quantum Photonics, University of Ottawa, 25 Templeton Street, Ottawa, Ontario K1N 6N5, Canada
(Received 26 April 2016; revised manuscript received 14 June 2016; published 12 September 2016)

"One drawback of conventional quantum state tomography is that it does not readily provide access to single density matrix elements since it requires a global reconstruction."

Direct measurement of the wave function

A previous work:



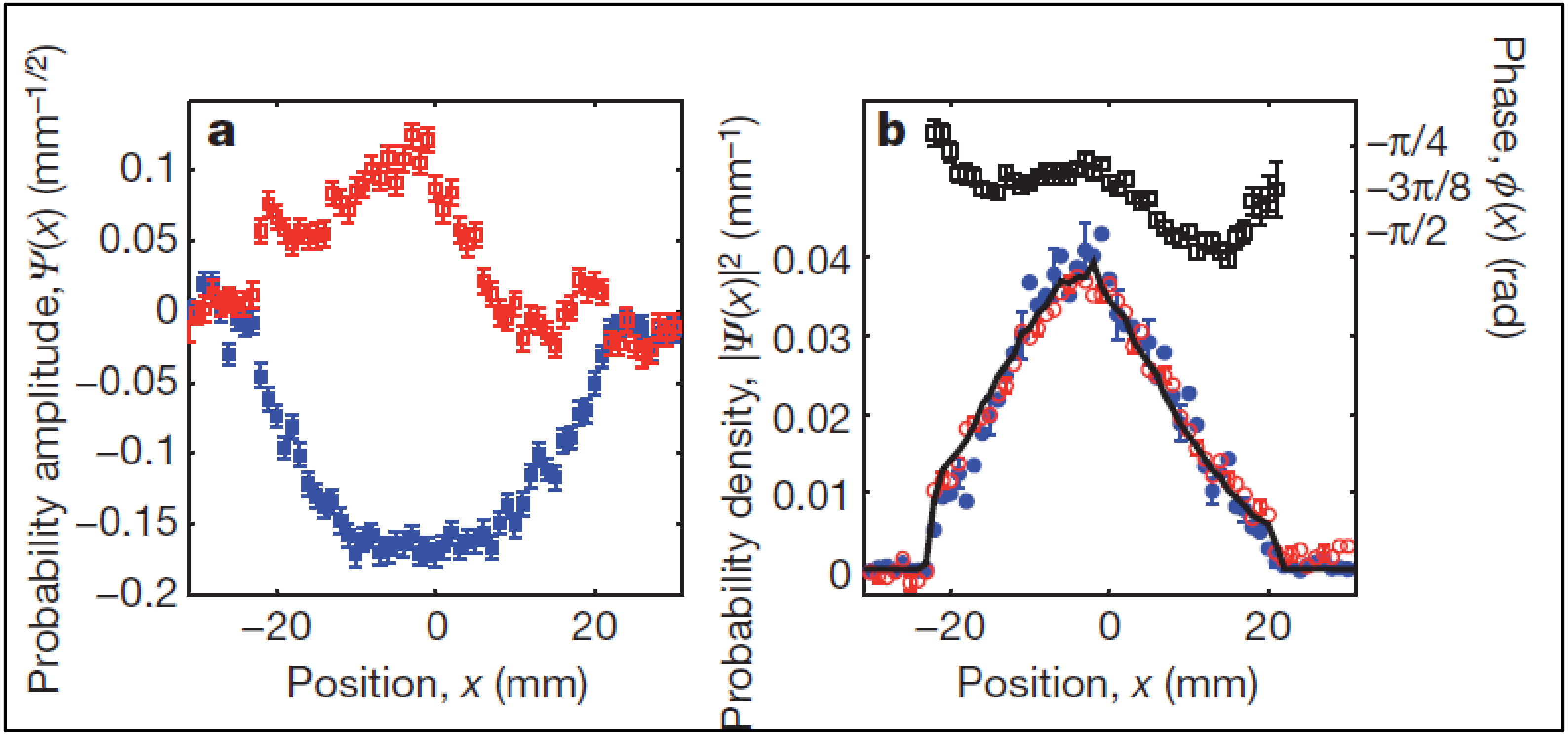
The complex nature of the weak value is what enables the authors to directly measure the real and imaginary parts of the wave function

$$|\Psi\rangle = \nu \sum_a \langle \pi_a \rangle_{\mathrm{w}} |a\rangle$$

[Lundeen J, et al. Nature **474**, 188 (2011)]

Direct measurement of the wave function

Single photon reconstructed quantum wave function:



[Lundeen J, et al. Nature **474**, 188 (2011)]

Direct measurement of the density matrix

Key ideas:

- "Weak average"

$$\langle \mathcal{C} \rangle_{\mathcal{S}} = \mathrm{Tr}_{\mathcal{S}}[\mathcal{C}\rho_{\mathcal{S}}] = (1/\delta)\left\langle \mathbf{q} + i2\sigma^2 \mathbf{q}/\hbar \right\rangle_{\mathcal{P}}$$

It differs from the weak value, there is no postselection

- Weak average and density matrix elements connection

Let **A** and **B** two observables maximally incompatible or "complementaty"

$$|\langle a_i | b_0 \rangle| = 1/\sqrt{d}$$

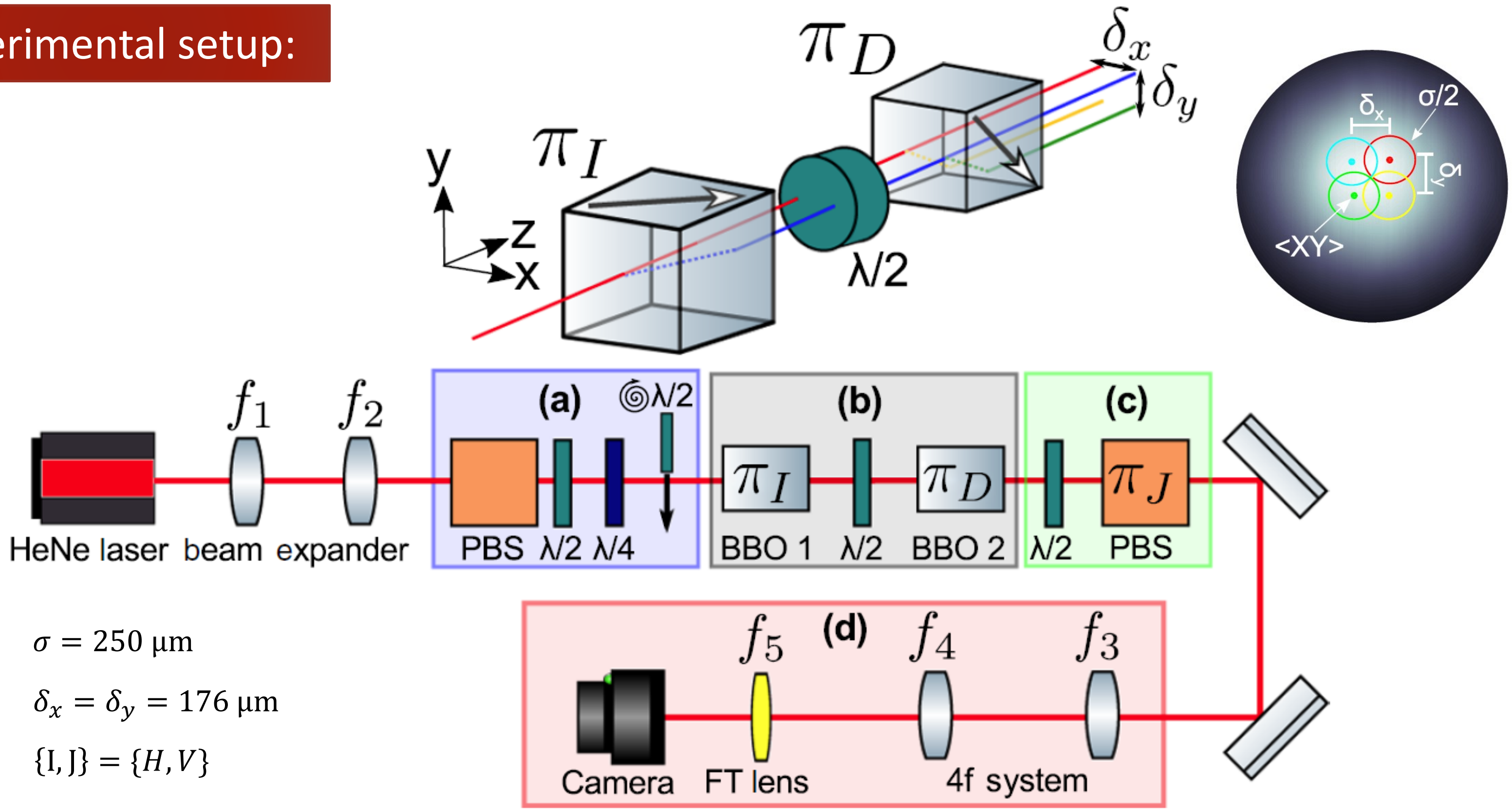
$$\Pi_{a_i a_j} = \pi_{a_j}\pi_{b_0}\pi_{a_i}, \quad \pi_{a_i} = |a_i\rangle\langle a_i|, \quad \pi_{b_0} = |b_0\rangle\langle b_0|,$$

$$\langle \Pi_{a_i a_j} \rangle_{\mathcal{S}} = \mathrm{Tr}_{\mathcal{S}}\left[\pi_{a_j}\pi_{b_0}\pi_{a_i}\rho_{\mathcal{S}}\right] = \rho_{\mathcal{S}}(i,j)/d$$

Can be replaced by a strong measurement without affecting the weak average

Direct measurement of the density matrix

Experimental setup:



$\sigma = 250\ \mu\text{m}$

$\delta_x = \delta_y = 176\ \mu\text{m}$

$\{I, J\} = \{H, V\}$

[G. S. Thekkadath et. al. PRL **117**, 120401 (2016)]

Direct measurement of the density matrix

Key ideas: The measurements

- The final connection: Join position and momentum expectation values with density matrix elements relations

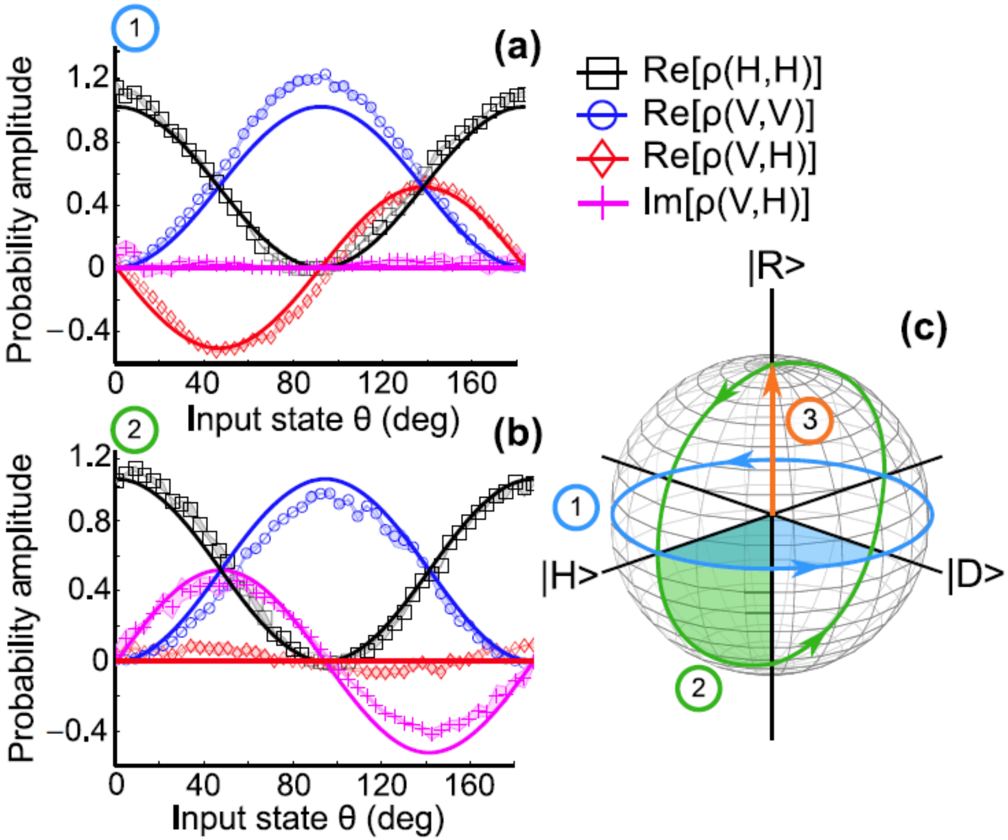
$$\mathrm{Re}[\rho_S(I,J)] = \frac{2}{\delta^2}\left(\langle \boldsymbol{x}_I \boldsymbol{y}_D \rangle_{\mathcal{P},J} - \frac{\sigma^2}{\sigma_p^2} \langle \boldsymbol{p}_{xI} \boldsymbol{p}_{yD} \rangle_{\mathcal{P},J} \right),$$

$$\mathrm{Im}[\rho_S(I,J)] = \frac{2}{\delta^2}\frac{\sigma}{\sigma_p}\left(\langle \boldsymbol{p}_{xI} \boldsymbol{y}_D \rangle_{\mathcal{P},J} + \langle \boldsymbol{x}_I \boldsymbol{p}_{yD} \rangle_{\mathcal{P},J} \right).$$

$$\iint xy\mathrm{Prob}(x,y,J)\,dxdy \equiv \langle \boldsymbol{xy} \rangle_{\mathcal{P},J}$$

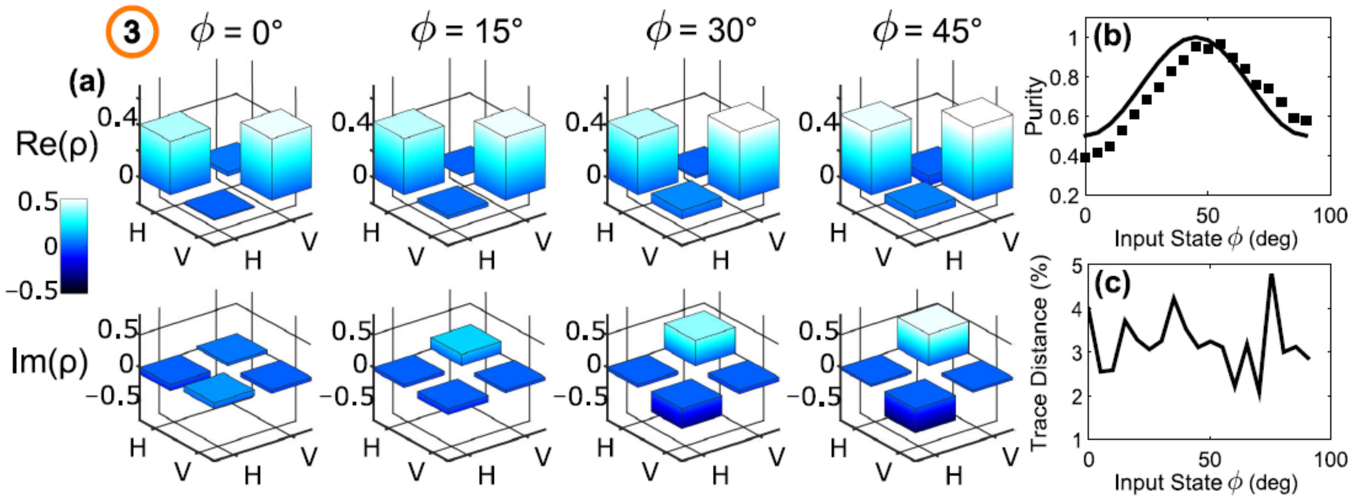
Direct measurement of the density matrix

Experimental results:
Pure states



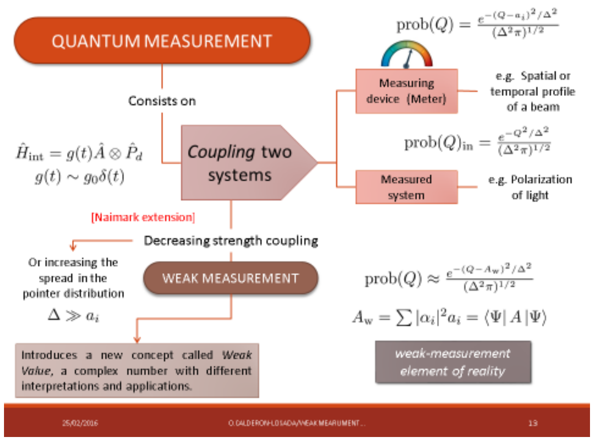
Direct measurement of the density matrix

Experimental results: Mixed states



$$\rho = \begin{pmatrix} 1/2 & i\sin\phi\cos\phi \\ -i\sin\phi\cos\phi & 1/2 \end{pmatrix},$$

In brief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$\hat{H}_{\text{int}} = g(t)\hat{A}\otimes\hat{P}_d$
$g(t) \sim g_0\delta(t)$
$\text{prob}(Q) = \frac{e^{-(Q-a_i)^2/\Delta^2}}{(\Delta^2\pi)^{1/2}}$
Measuring device (Meter)
e.g. Spatial or temporal profile of a beam
$\text{prob}(Q)_{\text{in}} = \frac{e^{-Q^2/\Delta^2}}{(\Delta^2\pi)^{1/2}}$
Measured system
e.g. Polarization of light
[Naimark extension]
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Or increasing the spread in the pointer distribution
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$\text{prob}(Q) \approx \frac{e^{-(Q-A_w)^2/\Delta^2}}{(\Delta^2\pi)^{1/2}}$
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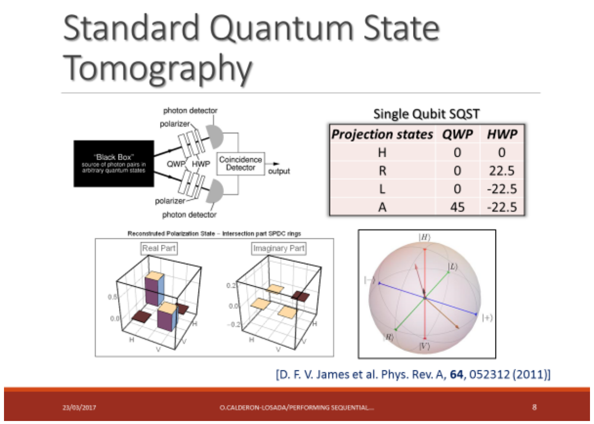
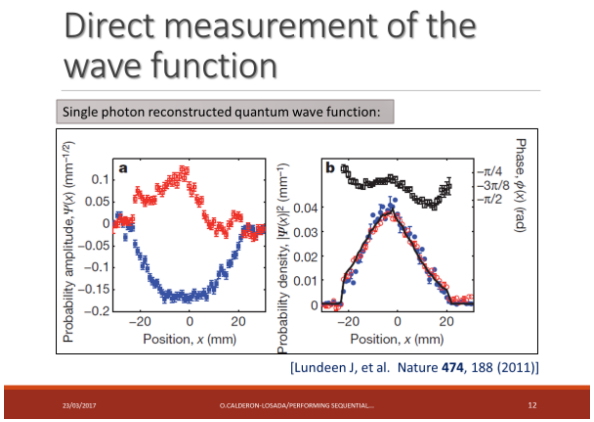
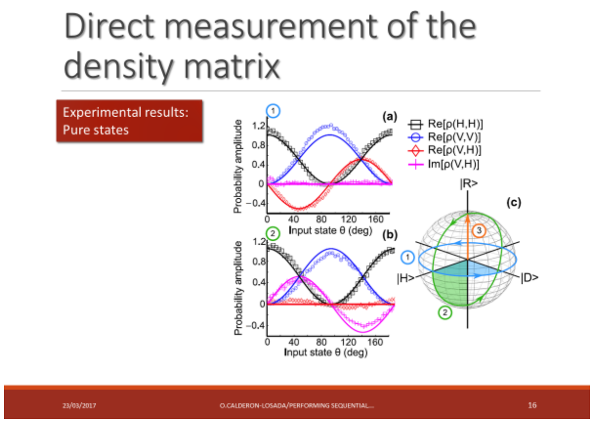

Standard Quantum State Tomography
Single Qubit SQST
Projection states | QWP | HWP
H 0 0
R 0 22.5
L 0 -22.5
A 45 -22.5
photon detector
polarizer
QWP HWP
Coincidence Detector
output
Real Part
Imaginary Part
[D. F. V. James et al. Phys. Rev. A, 64, 052312 (2011)]


Direct measurement of the wave function
Single photon reconstructed quantum wave function:
Probability amplitude, Ψ(x) (mm^-1/2)
Position, x (mm)
Probability density, |Ψ(x)|² (mm^-1)
Phase, ϕ(x) (rad)
[Lundeen J, et al. Nature 474, 188 (2011)]


Direct measurement of the density matrix
Experimental results: Pure states
Probability amplitude
Input state θ (deg)
Re[ρ(H,H)]
Re[ρ(V,V)]
Re[ρ(V,H)]
Im[ρ(V,H)]
|R>
|H>
|D>

THANKS FOR YOUR ATTENTION

AND YOUR QUESTIONS!

En resumen

Para medir directamente el estado de un sistema cuántico arbitrario:

Elegir una base $\{|a\rangle\}$ cualquiera asociada con un observable $\hat{A}$.

Medir débilmente un proyector de esta base, $\pi_a \equiv |a\rangle\langle a|$.

Realizar una post-selección en un valor b_0 asociado al observable $\hat{B}$ complementario a $\hat{A}$, es decir $[\hat{A}, \hat{B}] \neq 0$.

El estado se obtiene

$$\langle \pi_a \rangle_w = \frac{\langle b_0 | a \rangle \langle a | \Psi \rangle}{\langle b_0 | \Psi \rangle} = \langle a | \Psi \rangle / \nu$$

$$|\Psi\rangle = \nu \sum_a \langle \pi_a \rangle_w |a\rangle$$

¿Qué y cómo medimos esto?