Monitoring a Quantum Walk through Weak Measurements

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Contents

1. Motivation
2. Quantum Walk
   - How to implement it
   - Variance Behavior
3. Weak Values

Quantum Walk Search

- Quantum walk search algorithms

A Quantum Random Walk Search Algorithm

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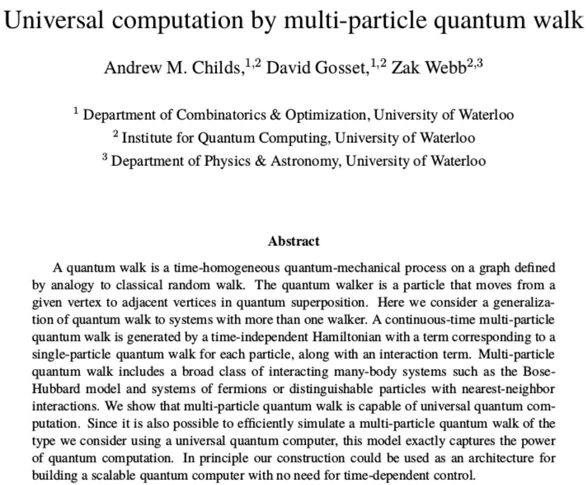
(Dated: February 1, 2008)

Quantum random walks on graphs have been shown to display many interesting properties, including exponentially fast hitting times when compared with their classical counterparts. However, it is still unclear how to use these novel properties to gain an algorithmic speed-up over classical algorithms. In this paper, we present a quantum search algorithm based on the quantum random walk architecture that provides such a speed-up. It will be shown that this algorithm performs an oracle search on a database of N items with $O\left(\sqrt{N}\right)$ calls to the oracle, yielding a speed-up similar to other quantum search algorithms. It appears that the quantum random walk formulation has considerable flexibility, presenting interesting opportunities for development of other, possibly novel quantum algorithms.


Universal computation by multi-particle quantum walk

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Abstract

A quantum walk is a time-homogeneous quantum-mechanical process on a graph defined by analogy to classical random walk. The quantum walker is a particle that moves from a given vertex to adjacent vertices in quantum superposition. Here we consider a generalization of quantum walk to systems with more than one walker. A continuous-time multi-particle quantum walk is generated by a time-independent Hamiltonian with a term corresponding to a single-particle quantum walk for each particle, along with an interaction term. Multi-particle quantum walk includes a broad class of interacting many-body systems such as the Bose-Hubbard model and systems of fermions or distinguishable particles with nearest-neighbor interactions. We show that multi-particle quantum walk is capable of universal quantum computation. Since it is also possible to efficiently simulate a multi-particle quantum walk of the type we consider using a universal quantum computer, this model exactly captures the power of quantum computation. In principle our construction could be used as an architecture for building a scalable quantum computer with no need for time-dependent control.

A tool for research

- Used as a tool for the modeling of phenomena as:

Energy transfer in the photosynthesis process.

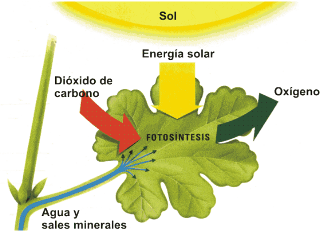

Figure: Taken from [1]

A tool for research

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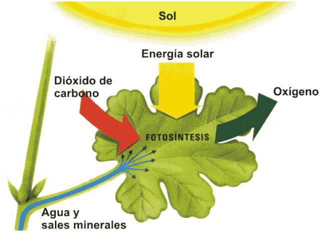


Figure: Taken from [1]

Topological quantum phase transition.

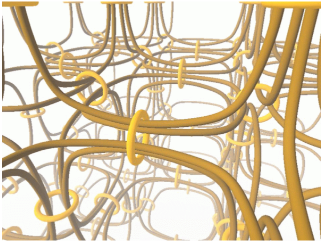
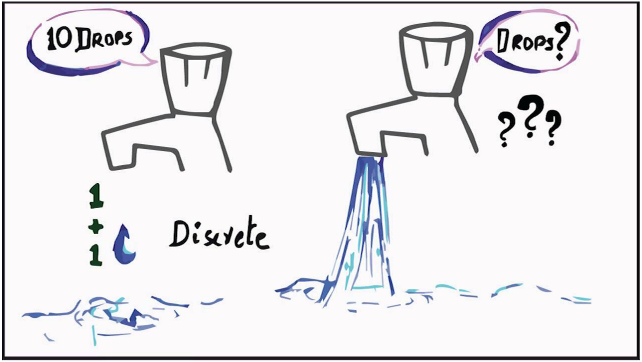


Figure: Taken from [2]

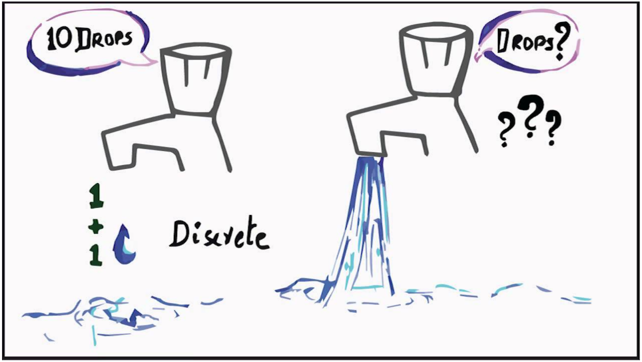
Quantum Walk

- Continuous time quantum walk
- Discrete time quantum walk



Quantum Walk

- Continuous time quantum walk
- **Discrete time quantum walk**



Discrete Time Quantum Walk

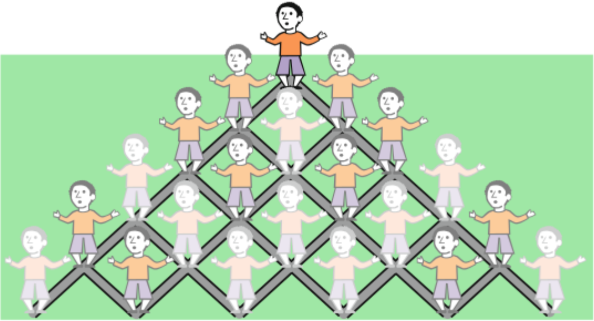


Figure: Taken from [3]

Discrete Time Quantum Walk

- Two degrees of freedom:
  - The walker.(Evolves in the "position" space)
  - The coin.

Discrete Time Quantum Walk

- Two degrees of freedom:
  - The walker.(Evolves in the "position" space)
  - The coin.
- Temporal evolution:
  - Unitary coin operator.("Throwing the coin")
  - Unitary conditional shift operator. (Depending on the coin state the walker moves in some direction)

The Walker

- "Lives" in a Hilbert space that will be called $\mathcal{H}_p$, whose dimension can be infinite.
- Important to note that the position does not necessarily correspond to the usual space.
- Examples of such spaces:
  - Angular momentum.
  - Actual position. $\vec{x}$

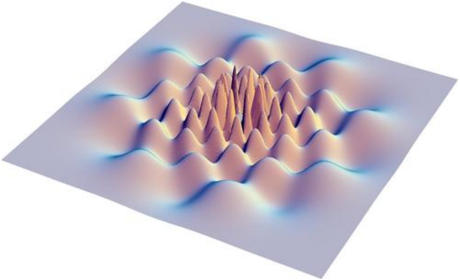
The Coin

- It belongs to a Hilbert space $\mathcal{H}_c$ and its state instructs which way the walker can move in the position space.
- In the one dimensional case only two possible ways are usually treated, going left or right, and thus $\mathcal{H}_c$ is spanned by two basis states: $\{|0\rangle_c, |1\rangle_c\}$.
- Examples of such space:
    - Spin.
    - Polarization.

Quantum Walk

- The complete quantum state belongs to the Hilbert space $\mathcal{H} = \mathcal{H}_c \otimes \mathcal{H}_p$.

$$|\Psi\rangle = (\cos(\theta/2)\,|0\rangle_c + \sin(\theta/2)e^{i\eta}\,|1\rangle_c) \otimes |\psi_{x0}\rangle_p$$



Coin

Genericallly:

$$C_{\zeta,\alpha,\beta,\gamma} = e^{\frac{i}{\hbar}\zeta} e^{\frac{i}{\hbar}\alpha\sigma_x} e^{\frac{i}{\hbar}\beta\sigma_y} e^{\frac{i}{\hbar}\gamma\sigma_z}$$

Hadamard:

$$\hat{H} \equiv \frac{1}{\sqrt{2}} \left(|0\rangle_c \langle 0| + |0\rangle_c \langle 1| + |1\rangle_c \langle 0| - |1\rangle_c \langle 1| \right)$$

Temporal Evolution

Coin

Generically:

$$C_{\zeta,\alpha,\beta,\gamma} = e^{\frac{i}{\hbar}\zeta} e^{\frac{i}{\hbar}\alpha\sigma_x} e^{\frac{i}{\hbar}\beta\sigma_y} e^{\frac{i}{\hbar}\gamma\sigma_z}$$

Hadamard:

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Conditional Shift

$$\hat{S} = e^{-\frac{i}{\hbar}\left(|0\rangle_c\langle 0| - |1\rangle_c\langle 1|\right)\otimes \hat{P} l}$$

Temporal Evolution

$$\hat{U} = \hat{S}\left(\hat{C} \otimes \mathbb{I}_p\right)$$

Outline

1 Motivation

2 Quantum Walk
- How to implement it
- Variance Behavior

3 Weak Values

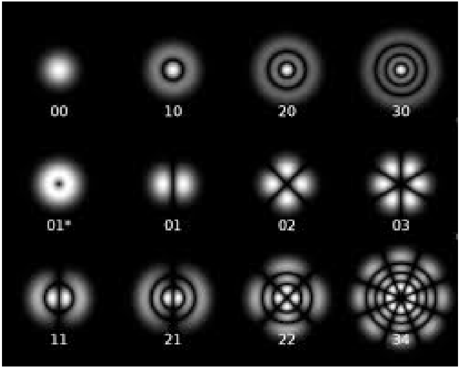
How to implement it

Optical implementation of one-dimensional quantum random walks using orbital angular momentum of a single photon

Xubo Zou, Yuli Dong and Guangcan Guo
Key Laboratory of Quantum Information, University of Science and Technology of China, Chinese Academy of Sciences, Hefei, Anhui 230026, People's Republic of China
E-mail: xbz@ustc.edu.cn

How to implement it

- **Degrees of freedom:**
  - Walker: Orbital Angular Momentum of photons.
  - Coin: Optical Path.



How to implement it

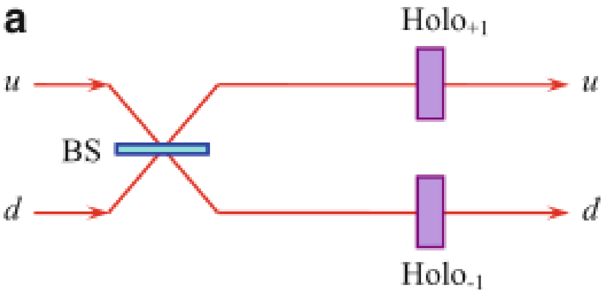


Figure: Taken from [3]

How to implement it

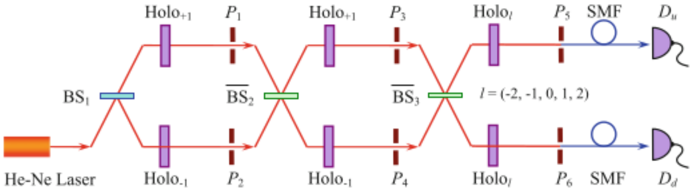


Figure: Taken from [3]

Not really scalable

How to implement it

- Based on [4].
- **Degrees of freedom:**
  - Walker: Horizontal axis belonging to the perpendicular plane to the photon beam's propagation direction.
  - Coin: Polarization.

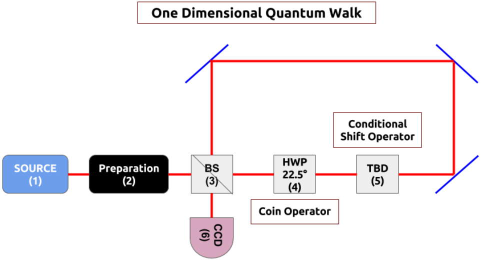
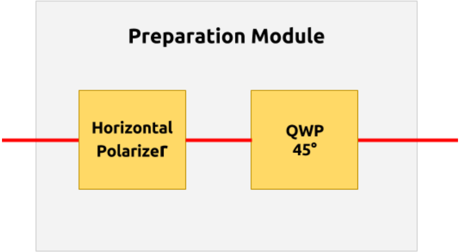


Figure: (1) Photons source , (2) Preparation module, (3) Beam Splitter, (4) Half Wave Plate with fast axis at an angle $\pi/8$ with respect to the horizontal, (5) Tunable Beam Displacer (TBD), (6) Charged Coupled Device Camera.

How to implement it



$$\hat{A}_{\lambda/4}(pi/4) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$|\Psi_{ini}\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_c + i\,|V\rangle_c \right) \otimes |\psi_{x0}\rangle_p$$

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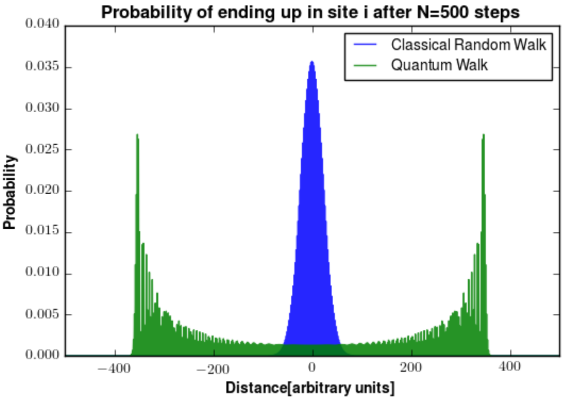
Considerations

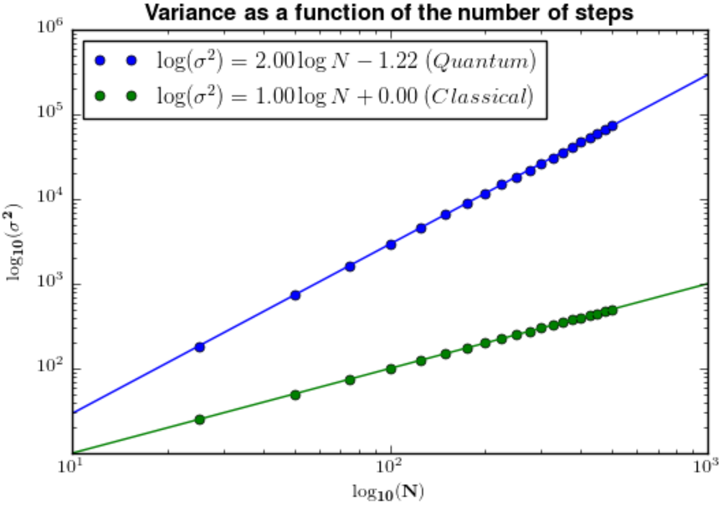
- No Overlap $\rightarrow \mathcal{H}_p = span(\{|i\rangle_p\})$ con $\langle i|j\rangle = \delta_{ij}$.
- $\hat{C} = \hat{H}$ (Hadamard).
- $|\Psi_{ini}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_c + i\,|V\rangle_c) \otimes |0\rangle_p$

Variance Behavior

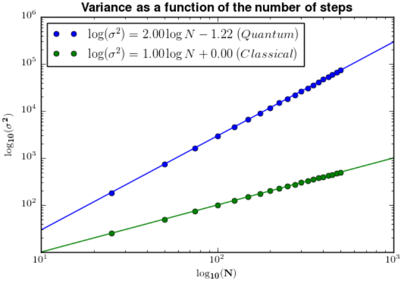
$$|\Psi_f\rangle = U^{500}|\Psi_{ini}\rangle$$
$$= \left[\hat{S}\left(\hat{H}\otimes\hat{\mathbb{I}}_p\right)\right]^{500}|\Psi_{ini}\rangle$$



Variance Behavior



Variance Behavior



Regime	Dependence on Steps
Quantum	$\sigma^2 = 0.3N^2$
Classic	$\sigma^2 = N$

Weak Values

- "Complex numbers that one can assign to the powers of a quantum observable operator $\hat{A}$ using two states: an initial state $|i\rangle$, called the preparation or preselection, and a final state $|f\rangle$, called the postselection."[5]
- The nth order weak value of $\hat{A}$ has the form:

$$A_w^n = \frac{\langle f|\,\hat{A}\,|i\rangle}{\langle f|i\rangle}$$

Weak Values

- **Prepare and Measure Experiment**
  - Initial state $|i\rangle$.
  - The probability of detecting $|f\rangle$.

$$P = |\langle f|i\rangle|^2$$

- **Modifying the procedure with an intermediate interaction**
  - $\hat{U}(\epsilon) = e^{-i\epsilon\hat{A}}$ (Stone's theorem).
  - Detection probability:

$$P_\epsilon = |\langle f|i'\rangle|^2 = |\langle f|\hat{U}(\epsilon)|i\rangle|^2$$

Weak Values

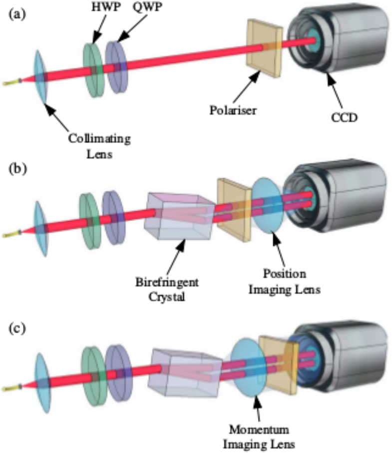
$$P_\epsilon = |\langle f|\hat{U}(\epsilon)|i\rangle|^2 = |\langle f|(1 - i\epsilon\hat{A} + ...)|i\rangle|^2$$
$$= P + 2\epsilon Im\{\langle i|f\rangle\langle f|\hat{A}|i\rangle\} + O(\epsilon^2)$$

- Getting the weak value.

$$\frac{P_\epsilon}{P} \approxeq 1 + 2\epsilon Im\{A_w\} + O(\epsilon^2)$$

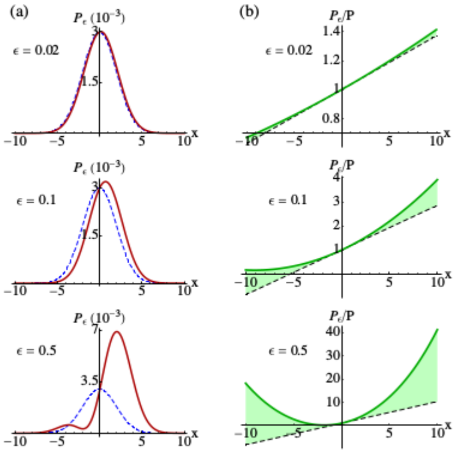
Weak Values

- How to measure it.



Weak Values

- How to measure it.



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