Reconstruction of single and 2-qubit density matrices using quantum state tomography

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Abstract

We report an experimental realization of quantum state tomography of a photonic ensemble for single and two-qubit polarization states. Our implementation is based on the work by James, Kwiat, Munro and White [3] which provides good tomographic reconstructions based on local projective measurements. We describe the theory and experimental tomographic measurement for single qubit states prepared from a laser source and present the tomographic reconstruction of the density matrices of three orthogonal bases. Furthermore, we also describe the theory and experimental realization of a quantum state tomography of the two polarization degrees of freedom of a pair of entangled photons generated in a down-conversion experiment. Two different techniques are discussed: a linear reconstruction, where the density matrix is constructed from coincidence measurements but with the caveat of possibly producing non-physical density matrices, and a maximum likelihood estimation technique that produces physical density matrices. Finally, we also discuss temporal compensation of the down-converted photons in type II BBO crystal and its effect on the tomographic reconstruction of 2-qubit states and present the tomographic reconstruction of the density matrix for an SPDC source.

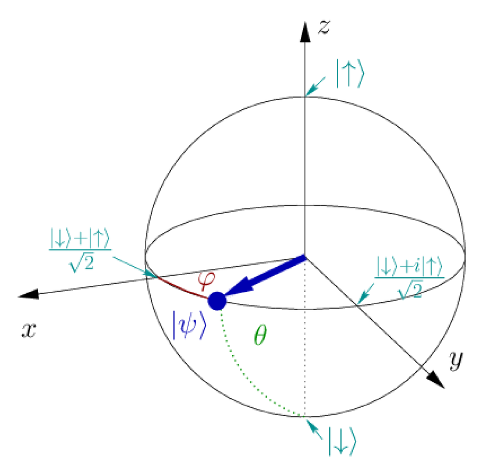
Motivation

Why should we be interested in performing a quantum state tomography (QST) to a photonic system? ⟶ To reconstruct the polarization density matrix, ρ, of the system!

1 Single qubit quantum state tomography

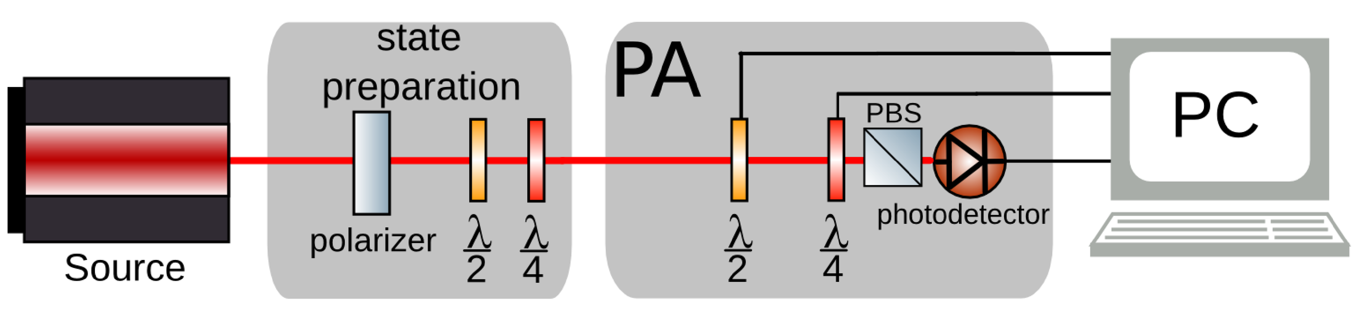
Photon polarization state vector

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|H\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\varphi}|V\rangle\,,$$



Two-level system → qubit.

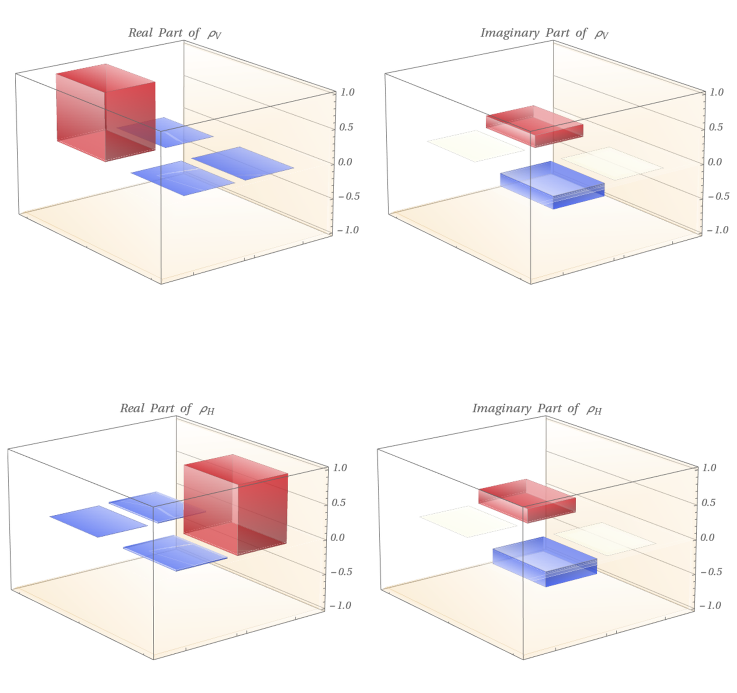
1.1 Experimental setup & results

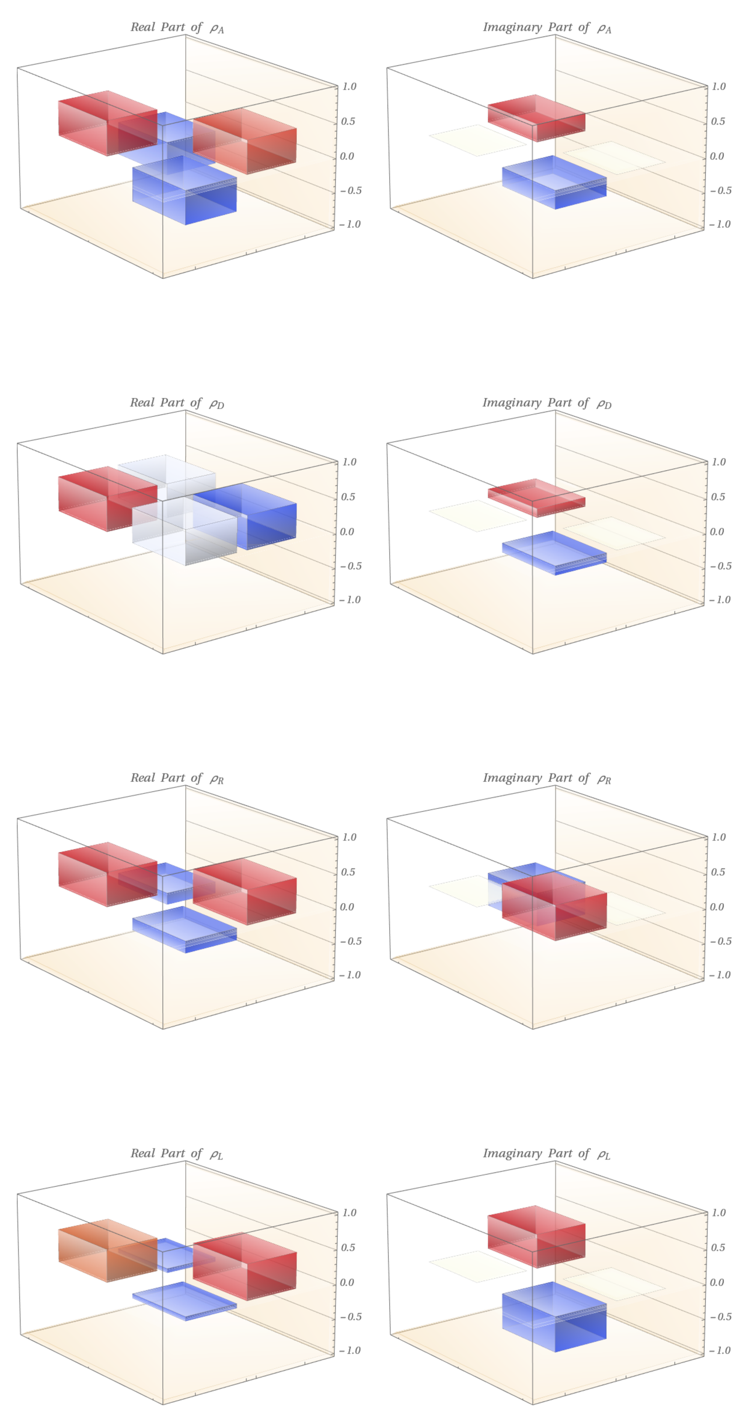


Single qubit tomographic reconstructions

Table 1: Angles h and q, in the HWP and QWP, to project a *vertical* state ρ_V in the polarization state $|i\rangle$.

State	$h\,(^o)$	$q\,(^o)$
$\|V\rangle$	0.0	0.0
$\|H\rangle$	45.0	0.0
$\|A\rangle$	22.5	0.0
$\|D\rangle$	337.5	0.0
$\|R\rangle$	0.0	315.0
$\|L\rangle$	0.0	45.0





2 Two-qubit quantum state tomography

The appropriate basis in which we should write our density matrix is known as the tomographic basis [2]. In this basis the density matrix is written as

$$\rho = \frac{1}{\mathcal{N}}\sum_{\mu=1}^{16} M_\mu n_\mu$$

- Hermitian ✓
- Normalized ✓
- Positive X $\xrightarrow{MLE}$ ✓

2.1 Experimental setup & results

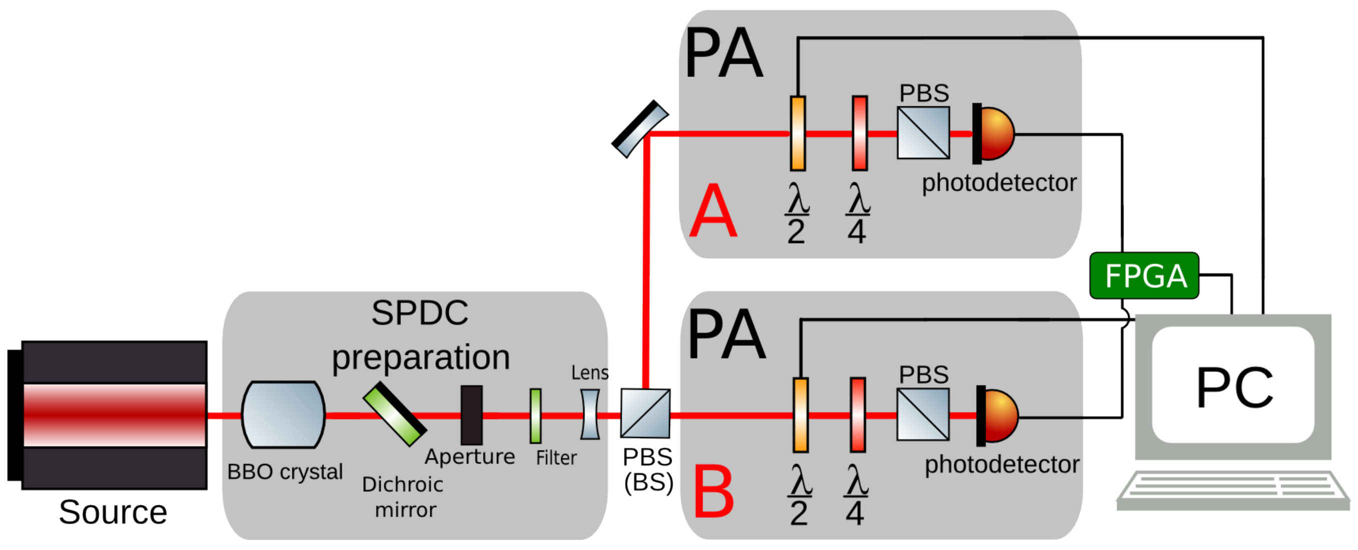
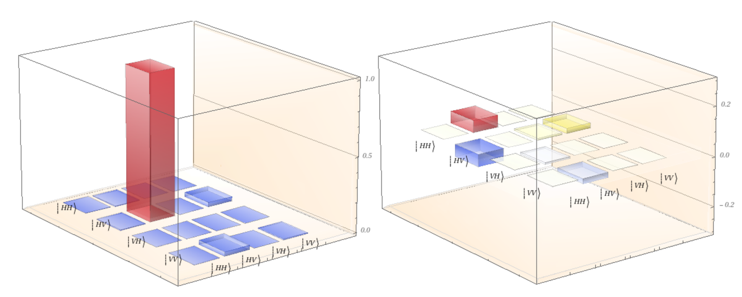


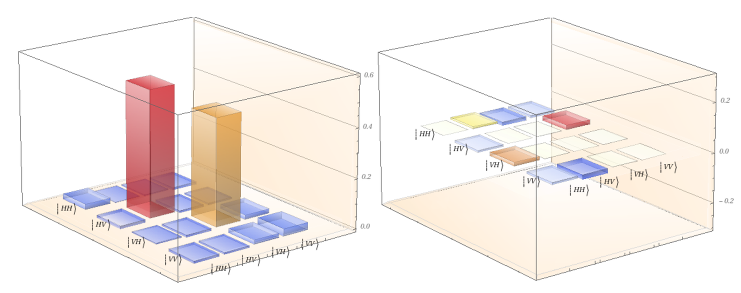
Table 2: Experimental procedure to project the prepared state into the 16 states of the tomographic basis. h_A and q_A stand for the angles in the HWP and QWP respectively in the *path A*, analogous for *path B*.

ν	A	B	$h_A\,(^o)$	$q_A\,(^o)$	$h_B\,(^o)$	$q_B\,(^o)$
1	$\|H\rangle$	$\|H\rangle$	45.0	0.0	45.0	0.0
2	$\|H\rangle$	$\|V\rangle$	45.0	0.0	0.0	0.0
3	$\|V\rangle$	$\|V\rangle$	0.0	0.0	0.0	0.0
4	$\|V\rangle$	$\|H\rangle$	0.0	0.0	45.0	0.0
5	$\|R\rangle$	$\|H\rangle$	22.5	0.0	45.0	0.0
6	$\|R\rangle$	$\|V\rangle$	22.5	0.0	0.0	0.0
7	$\|D\rangle$	$\|V\rangle$	22.5	45.0	0.0	0.0
8	$\|D\rangle$	$\|H\rangle$	22.5	45.0	45.0	0.0
9	$\|D\rangle$	$\|R\rangle$	22.5	45.0	22.5	0.0
10	$\|D\rangle$	$\|D\rangle$	22.5	45.0	22.5	45.0
11	$\|R\rangle$	$\|D\rangle$	22.5	0.0	22.5	45.0
12	$\|H\rangle$	$\|D\rangle$	45.0	0.0	22.5	45.0
13	$\|V\rangle$	$\|D\rangle$	0.0	0.0	22.5	45.0
14	$\|V\rangle$	$\|L\rangle$	0.0	0.0	22.5	90.0
15	$\|H\rangle$	$\|L\rangle$	45.0	0.0	22.5	90.0
16	$\|R\rangle$	$\|L\rangle$	22.5	0.0	22.5	90.0

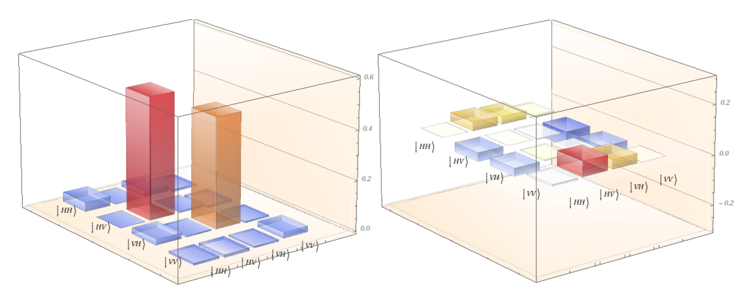
2-qubit tomography with PBS



2-qubit tomography with BS



2-qubit tomography with BS and Compensation Crystal



Conclusions

- By means of a single qubit quantum state tomography, we successfully reconstructed the density matrix for the six polarization states of the three different polarization bases.
- With two-qubit quantum state tomography and maximum likelihood estimation the density matrix of an SPDC source was reconstructed.
- The two-qubit density matrix reconstruction was made with a fidelity $\mathcal{F} = 0.974$ with a mixed and entangled density matrix.
- In type II BBO crystals temporal separation of the down-converted photons is an important factor to be considered since it causes decoherence of the photon pair.

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