Implementation of Computer Generated Holograms in a low-cost phase-only Spatial Light Modulator for modifying Light spatial distribution

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I. INTRODUCTION

Quantum computing aims to outperform classical computing in certain tasks, traditionally using two-level systems called qubits. Recently, higher-dimensional systems known as qudits have gained attention due to advantages like greater information density, increased noise resistance in quantum key distribution, and improved efficiency in some algorithms [1–3]. Light, with its various degrees of freedom—such as orbital angular momentum (OAM)—can be used to implement qudit-based quantum computing. Manipulating OAM requires applying quantum logic gates, which is made possible by holograms. While earlier methods involved physical holograms, modern setups use Spatial Light Modulators (SLMs) to dynamically project any hologram, allowing precise control of OAM and enabling optical quantum logic operations [4].

II. METHODOLOGY

A. Optical setup

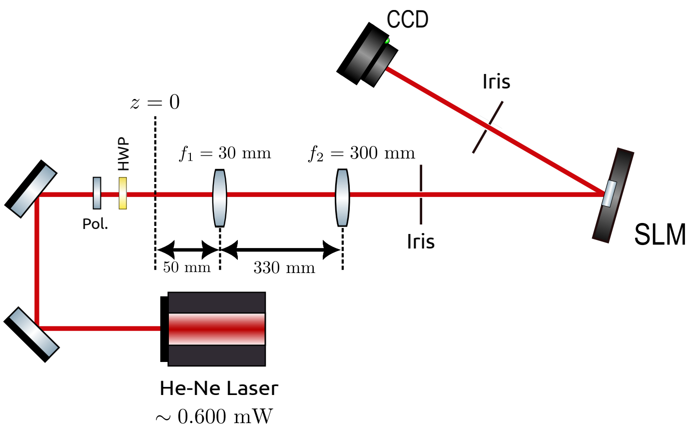

Figure 1. Optical setup implemented in Q402 for the generation and manipulation of OAM light beams.

The setup shown in Figure 1 is used for generating light beams carrying OAM and transforming them. A He-Ne laser (633 nm) is used as light source with around 0.6 mW of power. Then, as the SLM uses a definite polarization, the light beams pass through a polarizer and a HWP. After, the beam passes through a telescope system conformed by f_1 and f_2 which collimates and increases the size of the light beam. The light is filtrated by the iris and then reflected by the SLM, on which a hologram is projected. A phase delay is induced on the beam and, therefore, induces OAM on the light beam. The beam now passes through another iris so higher diffraction orders are filtered and then it is measured with a CCD Camera.

B. Spatial Light Modulators

Spatial Light modulators are composed of a series of layers of liquid crystals whose orientations can be changed depending on an external voltage. Most commonly, they are composed of twisted nematic (TN) microdisplay cells, in which the liquid crystals exhibit birefringence, making the refractive index in one axis higher than the other [5, 6]. Thus, by progressively changing the orientation of each layer, the relative phase of the light beam can be modified. This means that the light reflected of the SLM acquires a phase $\psi(x, y)$ that can be loaded into the SLM:

$$E_{\mathrm{SLM}}(x, y) = E_{\mathrm{in}}(x, y)e^{i\psi(x,y)} \tag{1}$$

C. Generation of Light Beams Carrying OAM Using SLMs

To recreate an arbitrary complex optical field, it is generally necessary to modulate both the amplitude and phase of the incident light. However, since phase-only SLMs are more readily available, it becomes essential to design holograms that encode amplitude information into the phase modulation.

This is achieved by structuring the hologram so that the desired complex field appears in a specific diffraction order. By controlling the amount of light directed into the zeroth order, one can indirectly modulate the amplitude of higher diffraction orders [7].

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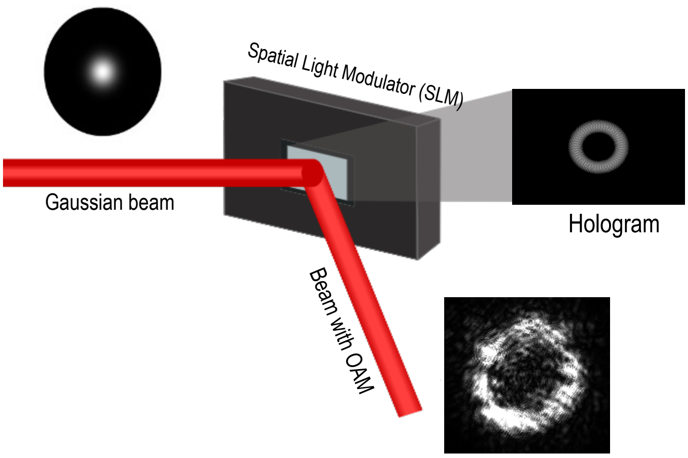

Figure 2. Diagram illustrating the generation of arbitrary optical fields using an SLM. A Gaussian beam illuminates the SLM, and the desired field appears in the first diffraction order of the reflected beam.

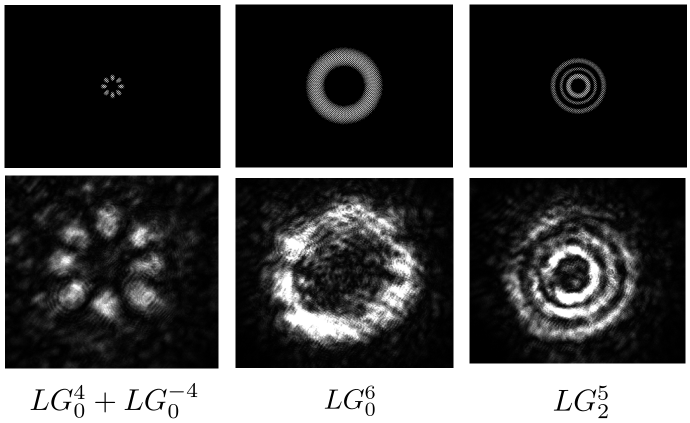

Figure 3. Results obtained for the generation light beams carrying OAM. Here, holograms projected on the SLM (first row) and output light beam (second row) are shown for different Laguerre-Gauss modes (LG_p^ℓ) and linear combination of them.

Following the method described by Bolduc *et al.* [8], we aim to generate a field of the form:

$$E_{\text{out}}(r,z) = \mathcal{A}(r,z)\, e^{i\Phi(r,z)},$$

in the first diffraction order. To achieve this, we use holograms defined as:

$$\Psi(m,n) = \mathcal{M}(m,n) \bmod \left(\mathcal{F}(m,n) + \frac{2\pi m}{\Lambda} + \frac{2\pi n}{\Lambda},\, 2\pi\right), \tag{2}$$

where m and n index the pixels on the SLM, Λ is the grating period (chosen diagonally for easier isolation of the first order), $\mathcal{M}(m,n)$ is a function of $\mathcal{A}(m,n)$, and $\mathcal{F}(m,n)$ is a function of both $\mathcal{A}(m,n)$ and $\Phi(m,n)$. This generalizes traditional kinoforms, which arise from interference between the desired field and a tilted plane wave [9]. The resulting modulated field is:

$$T(m,n) = e^{i\mathcal{M}(m,n) \bmod \left(\mathcal{F}(m,n) + \frac{2\pi m}{\Lambda} + \frac{2\pi n}{\Lambda},\, 2\pi\right)}.$$

Matching this term with the target field leads to the following exact expressions:

$$\mathcal{M}(m,n) = 1 + \frac{1}{\pi}\,\mathrm{sinc}^{-1}\left(\mathcal{A}(m,n)\right), \tag{3}$$

$$\mathcal{F}(m,n) = \Phi(m,n) - \pi\,\mathcal{M}(m,n), \tag{4}$$

where $\mathrm{sinc}^{-1}(x)$ denotes the inverse of $\sin(x)/x$.

Due to the computational complexity of this inverse, we instead use an approximate solution:

$$\mathcal{M}(m,n) = \frac{\mathcal{A}(m,n)}{\max(\mathcal{A}(m,n))},$$

$$\mathcal{F}(m,n) = \Phi(m,n) - \pi\,\mathcal{M}(m,n),$$

which is then used in equation (2) to produce grayscale (0–255) phase-only holograms loaded onto the SLM.

We tested the holograms by making Laguerre-Gauss modes of light, and superpositions of them. The LG_p^ℓ mode is generated by the expression:

$$E(r,\phi,z) = C_{lp}^{LG}\frac{1}{w(z)}\left(\frac{r\sqrt{2}}{w(z)}\right)^{|l|} \times \exp\left(-\frac{r^2}{w^2(z)}\right) L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right) \times \exp\left(-ik\frac{r^2}{2R(z)}\right)\exp(-il\phi)\exp(i\psi(z)) \tag{5}$$

Where L_p^l are the generalized Laguerre polynomials and C_{lp}^{LG} is a normalization constant.

Figure 3 shows results obtained, images captured by a CCD Camera and their corresponding hologram projected on the SLM. Images obtained with the camera match with those expected by theory. The size of the light beams and the contrast of the hologram can be adjusted with the code made for their generation. We wanted to know how much detail could be obtained by our SLM, in order to solve this question more complex hologram patterns were projected on the SLM, which are shown in Figure 4. We can see that small details like the ones in Figure 4a are not clearly resolved.This is product of the fact that the method used is approximate, and the limitations of resolution and phase modulation from the SLM.

D. Implementation of high-dimensional Z gate

For implementing a Z-gate, the optical setup shown in Figure 5. Literature suggests that Dove Prism by itself is capable of performing the exact same transformation made by a Z gate in high-dimensional spaces [10]. With the setup shown in Figure 5, it was seen that it does rotate OAM light beams by 2α when Dove Prism is rotated at α angle.

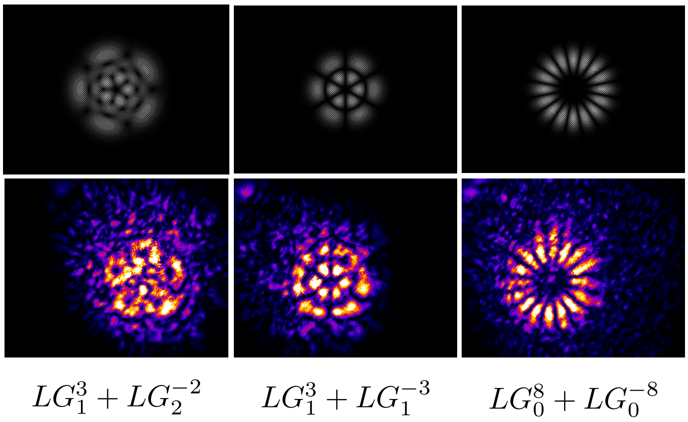

Figure 4. Results obtained for the generation of light beams carrying OAM with more complex hologram patterns. The figure shows the holograms projected on the SLM (first row) and their corresponding image measured with the CCD camera.

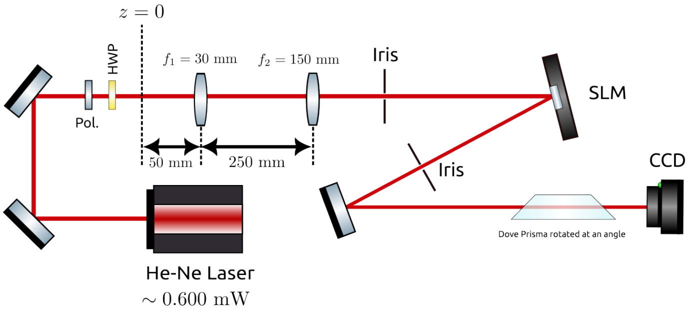

Figure 5. Setup for transforming OAM of light via a Dove Prism.

III. FUTURE WORK

Throughout the project, Laguerre-Gauss beams were generated by using an optical setup with a Spatial Light Modulator using the process described by Bolduc *et al.* [8]. Additionally, an experimental setup (Figure 5) was built in order to manipulate OAM so it performs a generalized Z-gate operation on the light beam.

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