Final Report – Experimental Project 202520: Experimental Observation of Integer and Fractional Talbot Effects

Samuel M. Hernández Caballero*

Universidad de los Andes

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This report summarizes the experimental work conducted during the semester in order to observe and characterize both integer and fractional Talbot effects. We begin by introducing the basic theoretical framework behind self-imaging in periodic structures. Then, we present a detailed characterization of the Gaussian beam used for illumination of the grating, discussing the limitations imposed by its propagation profile. Finally, we describe the choice of grating parameters, our measurement strategy, and the results obtained for both integer and fractional Talbot distances, emphasizing the experimental challenges and outlining improvements for future work.

I. INTRODUCTION

In 1836, Lord Talbot reported the surprising phenomenon of *self-imaging* that occurs when a periodic structure is illuminated by a coherent monochromatic field. Under constant amplitude plane-wave illumination, Fresnel diffraction predicts that at specific distances behind a grating, the transmitted field reproduces the grating's own transmittance function. These planes, known as *Talbot planes*, appear at integer multiples of the Talbot distance [1],

$$z_T = \frac{\ell^2}{\lambda},$$

where ℓ is the grating period and λ the wavelength of illumination.

At distances $z = \frac{p}{q} z_T$, with p, q coprime integers, the intensity pattern splits into q shifted replicas of the grating structure, each weighted by a phase factor determined by Gauss sums [2]. A schematic visualization of both cases is shown in Fig. 1.

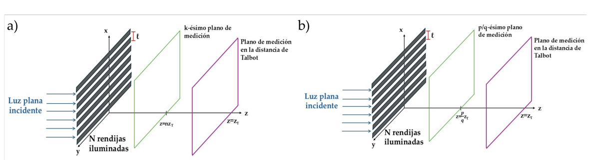

FIG. 1. Illustration of the integer (a) and fractional (b) Talbot effects.

Our goal in this project was to measure both effects experimentally using the setup shown in Fig. 2. Throughout the experiment, the illumination was provided by the **V-profile waist** of our laser beam (as obtained from the Gaussian beam characterization). Here, the V and W waists simply refer to the horizontal and vertical intensity profiles of the Gaussian beam, respectively. They are essentially 1D cuts of how the light spatially spreads in each direction. In our case, we used the horizontal (V) waist as the illumination region. After the lens system, approximately 27 slits of the grating were effectively illuminated. This number is important, as the formation of Talbot images strongly depends on the number of illuminated grating periods.

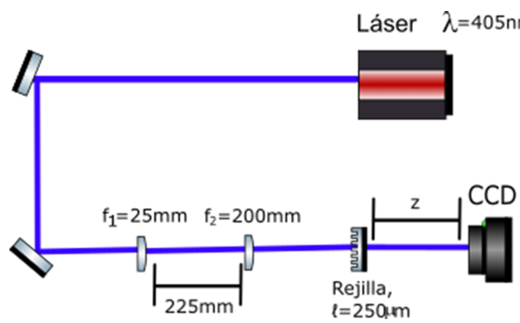

FIG. 2. Experimental setup used for observing integer and fractional Talbot effects.

II. GAUSSIAN BEAM PROPAGATION

A central part of the project was understanding and characterizing the Gaussian beam used as illumination. The transverse intensity profile of a Gaussian beam is [3]

$$I(r, z) = I_0 \exp\left[-\frac{2r^2}{\omega(z)^2}\right],$$

where $\omega(z)$ is the beam radius at distance z from the waist. The minimum radius is the beam waist ω_0, located at $z = z_0$. The Rayleigh range,

$$z_R = \frac{\pi \omega_0^2}{\lambda},$$

defines the region in which the beam retains an approximately constant radius.

Figure 3 shows the general behavior of a Gaussian beam. In our experiment, we used a 405 nm diode laser

* sm.hernandezc1@uniandes.edu.co

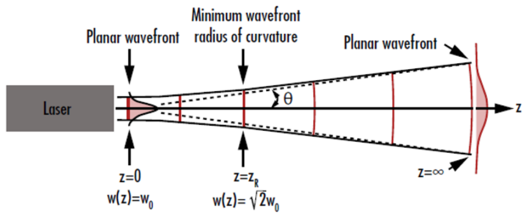

FIG. 3. Gaussian beam spatial parameters: waist ω_0, Rayleigh range z_R, and divergence θ. Taken from [3]

(Crystalaser), characterized with a Coherent BeamMaster. The measurement range extended from $z = 0$ to $z = 55\,\mathrm{cm}$.

Figure 4 shows the fitted $\omega(z)$ curves for both the W and V profiles. The illumination used in the Talbot experiment corresponds to the V-profile waist.

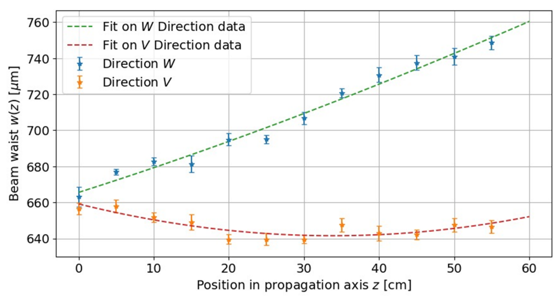

FIG. 4. Beam waist evolution $\omega(z)$ for W and V profiles. V-profile fit: $\omega_0 = 643.59 \pm 1.46\,\mu\mathrm{m}$, $z_0 = 584.91 \pm 2\,\mathrm{mm}$, $z_R = 321.308 \pm 0.18\,\mathrm{cm}$. W-profile fit: $\omega_0 = 536.64 \pm 102.19\,\mu\mathrm{m}$, $z_0 = -1637.33 \pm 11.08\,\mathrm{mm}$, $z_R = 223.4 \pm 0.5\,\mathrm{cm}$.

The fitted parameters show that the beam is very collimated in both profiles because the Rayleigh range is far larger than the dimensions of our setup. However, the fitted z_0 for the W profile is suspicious—its value suggests that the waist would lie outside our optical path. This inconsistency highlights the need for improved beam characterization. Moreover, since the Talbot effect assumes plane-wave illumination, the Gaussian profile is a plausible limiting factor in our observations, though this remains a hypothesis that must be tested.

III. TALBOT EFFECT MEASUREMENTS

A. Choice of Grating

To observe self-imaging, we used a Ronchi grating, whose Fourier coefficients are [2]:

$$g_0 = \frac{1}{2}, \qquad g_{2k} = 0, \qquad g_{2k+1} = \frac{(-1)^k}{\pi(2k+1)}.$$

We selected a period of $\ell = 250\,\mu\mathrm{m}$ (4 lines/mm). After the beam expansion optics, roughly 27 grating periods were illuminated.

B. Integer Talbot Effect

Figure 5 compares theory with experiment for the integer Talbot distance. The experiment shows the correct number of peaks (20) and a measured average spacing $\Delta x_{\mathrm{Av}} = 255.49\,\mu\mathrm{m}$ close to the theoretical value.

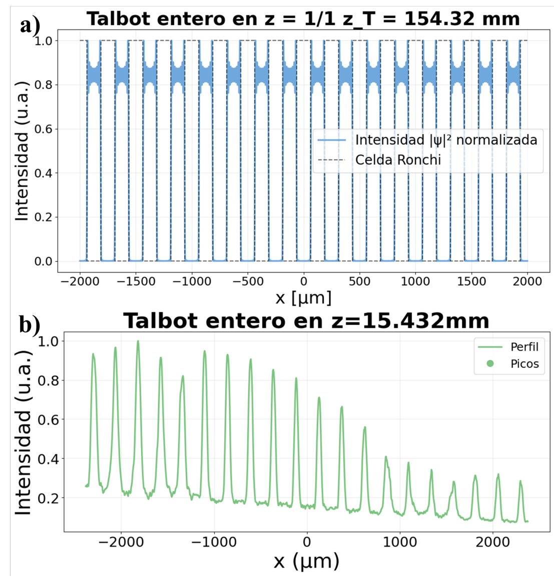

FIG. 5. Integer Talbot effect: theoretical a) vs experimental intensity profile b).

However, the expected "square-like" replication of the grating is not clearly visible. One possible explanation is the non-uniform illumination produced by the Gaussian beam rather than a plane wave, a hypothesis we aim to verify with improved modeling and measurements. The theoretical expression used to model fractional distances,

$$\psi_N\left(x, \tfrac{p}{q} z_T\right) = \sum_{s=-n_N}^{n_N} e^{\delta_s(x)} \frac{1}{\ell} \left(e^{-2\pi s x/\ell} * f(x/\ell)\right),$$

may yield distortions when the input field deviates from uniform illumination, something that will need to be explicitly tested numerically.

C. Fractional Talbot Effect

For the fractional distance $z = \frac{1858}{3858} z_T$, Fig. 6 compares theory and experiment. The smallest theoretical peak separation is around $2\,\mu\mathrm{m}$, while our CCD pixel size is $4.65\,\mu\mathrm{m}$. Therefore the CCD cannot resolve such structures.

Even though fine features cannot be resolved, the experimental pattern still shows the global periodicity $\ell = 250\,\mu\mathrm{m}$. A magnification stage must be added in future work.

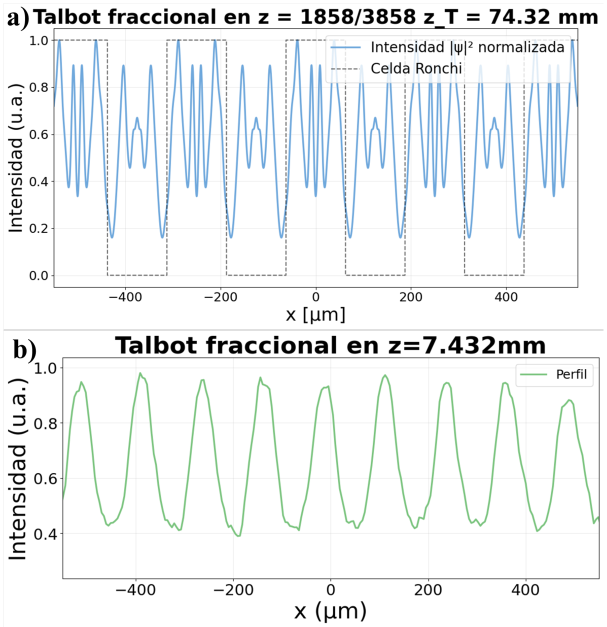

FIG. 6. Fractional Talbot effect: theory a) vs experiment b) for $z = \frac{1858}{3858} z_T$.

IV. CONCLUSIONS

In this project we experimentally explored both integer and fractional Talbot effects using a 405 nm diode laser and a Ronchi grating of 250 µm period. The integer effect was partially resolved. That is, we showed the correct number of peaks and nearly correct spacing. However, the expected self-imaging structure did not appear with the clarity predicted by theory. A plausible explanation is the non-uniform (Gaussian) illumination profile of the beam, but at this stage this remains a working hypothesis rather than a confirmed cause.

The fractional Talbot effect presented even stronger challenges: the theoretical sub-micron periodicities lie far below the CCD pixel size, making fine-structure resolution impossible under our current configuration. However, the global periodicity of $\ell = 250\,\mu$m was recovered experimentally.

Future work should focus on verifying whether the Gaussian illumination is responsible for deviations at integer Talbot distances, improving the Gaussian-beam characterization (particularly the inconsistent z_0 parameter for the W profile), adding a telescopic magnification stage to match CCD resolution with the fractional Talbot frequencies, and implementing a full numerical simulation including the real illumination profile via Fourier-space convolution methods.

[1] J. W. Goodman, *Introduction to Fourier Optics*, 2nd ed., McGraw-Hill Series in Electrical and Computer Engineering (McGraw-Hill, New York, 1996).

[2] M. V. Berry and S. Klein, Journal of Modern Optics **43**, 2139 (1996).

[3] Gaussian beam propagation, Application Note, Edmund Optics Laser Optics Resource Guide (n.d.), edmund Optics, urlhttps://www.edmundoptics.com/knowledge-center/application-notes/lasers/gaussian-beam-propagation.