Reconstruction of the Wigner function for a cat state using a classical Gaussian Beam

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Abstract

The Wigner function is a quasi-probability function that allows representing a quantum state in a phase space. Therefore, solving this function either analytically or experimentally enables us to identify behavior patterns of a specific state and, thereby, find ways to represent Wigner functions in the laboratory. In this work, it is demonstrated that the Wigner function of a cat state can be reconstructed using fractional Fourier transforms of two Gaussian beams, with these transforms serving as the marginal distributions of the Wigner function.

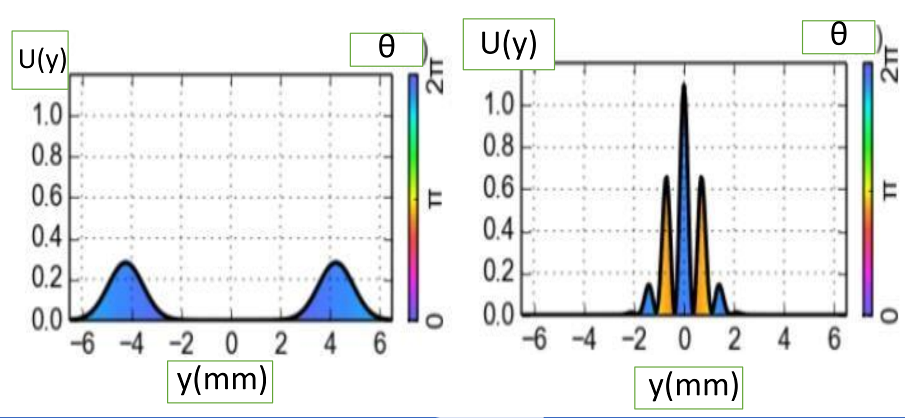
Cat state and Wigner Function

Cat state: Is a quantum state composed of two opposite coherence states of a single optical mode at same time :

$$|\text{cat}_e\rangle \propto |\alpha\rangle + |-\alpha\rangle$$

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

Two Gaussian Beam $u(y) = N(e^{-a(y-d)^2} - e^{-a(y+d)^2})$,



Wigner function of the state $\Psi(y)$

$$W(y,q_y) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}\psi^*\left(y-\frac{y'}{2}\right)\psi\left(y+\frac{y'}{2}\right)e^{-i\frac{y'q_y}{\hbar}}dy'$$

Results:

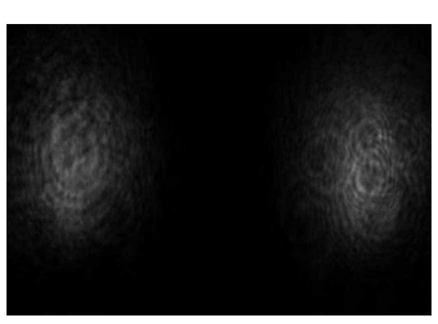
$$W_u(y,q_y) = \frac{1}{2\pi}N^2\sqrt{\frac{2\pi}{a}}e^{-q_y^2/2a}[e^{-2a(y-d)^2} + e^{-2a(y+d)^2} - 2e^{-2ay^2}\cos(2dq_y)].$$

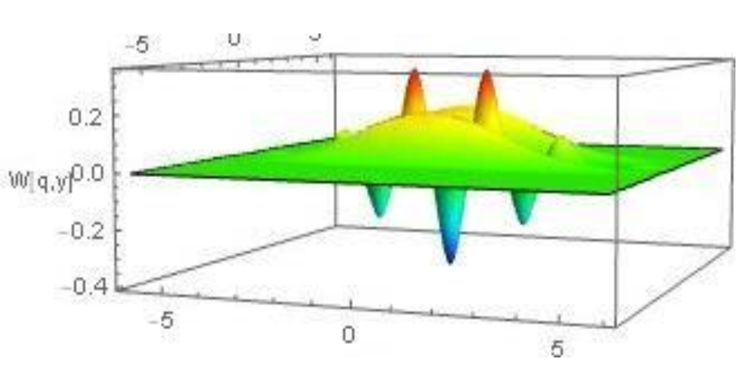
$$W_{cat}(y,q_y) = \frac{1}{2\pi}N^2\sqrt{4\pi}e^{-q_y^2}[e^{-(y-\tilde{y}_0)^2} + e^{-(y+\tilde{y}_0)^2} - 2e^{-y^2}\cos(2\tilde{y}_0 q_y)].$$

FFT and Wigner Function

FFT: Is a function composed by a number (n) of consecutives Fourier transform $F^{\{n\}}[g(t)] = FoFoFo\ldots\ldots Fo[g(t)]$ and represents a rotation for the function g(t) of $\theta = a\pi/2$ in the phase space.

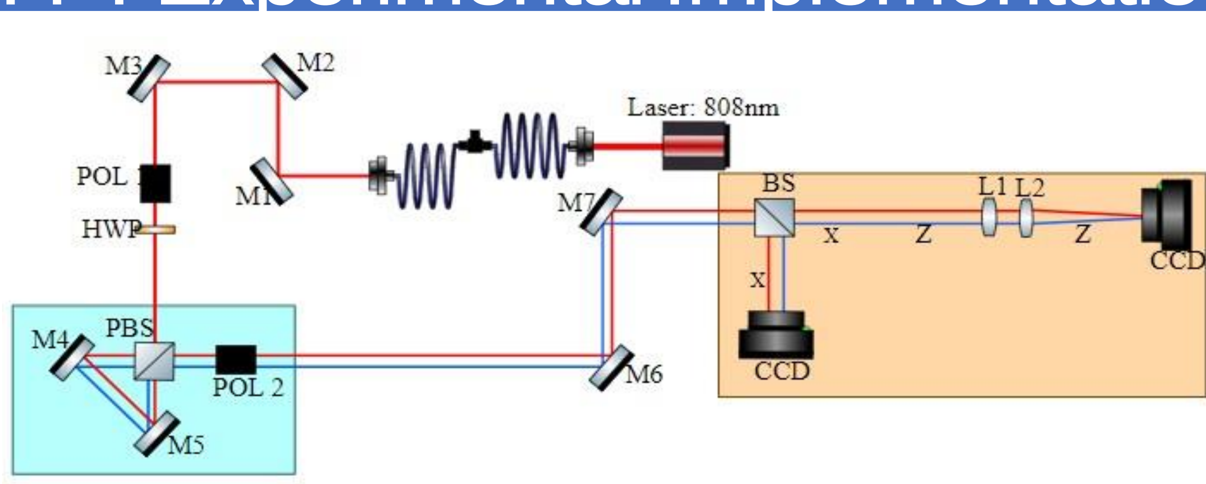
Marginal distribution: Is a Radon integral transform $R[W_E]$ and represents a projection on a plane $y = 0$ of a density function.





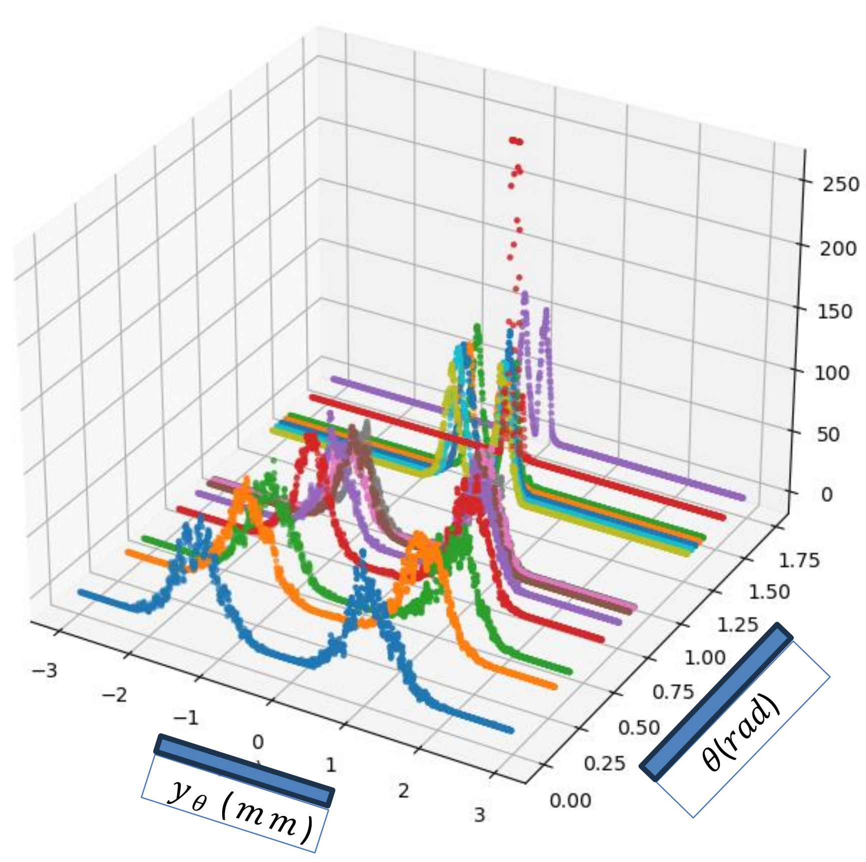
Signal E(x) → $F^{\{a\}}[|E(y)|^2]$ ↔ $R_{\{\theta\}}[W_E]$ ← $[W_E(x)] = [W(y,q_y)]$

FFT Experimental Implementation



- **Composed lens:** $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

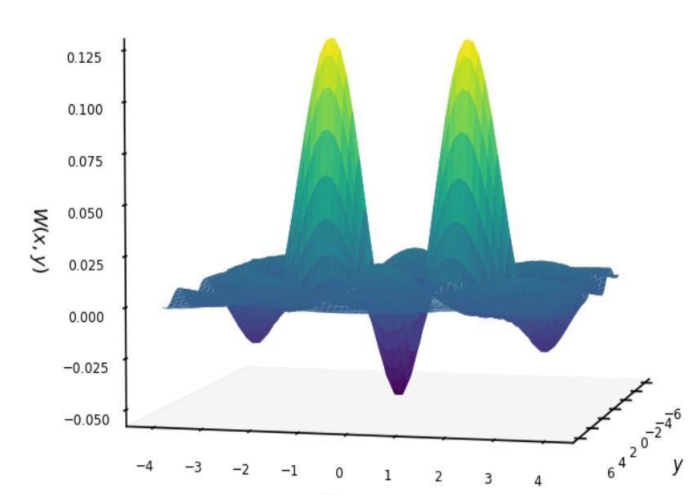
Results



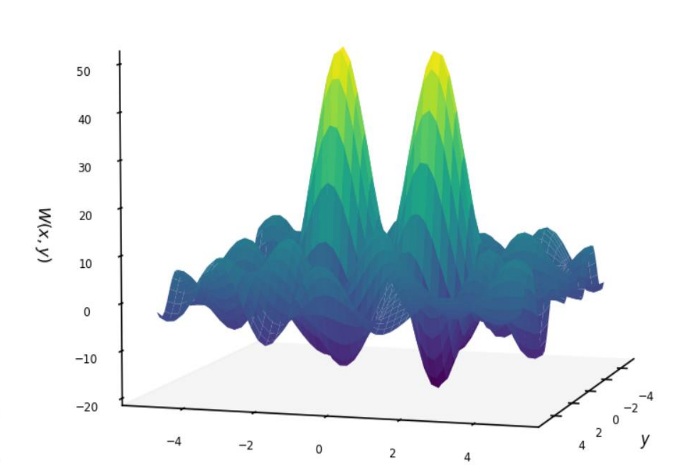
FFT measurements in the lab. y_θ is the conjugate variable of y. θ is the projection's angle of marginal distribution and $|F^n u\, y|$ is the module squared of FFT

- **Inverse Radon Transform:** Integral transform that uses the projection data obtain as the output as a tomography scan for remake an unknown function density

$$W(y,q_y) = \frac{1}{2\pi^2}\int_0^{\pi}\int_{-\infty}^{\infty}R_\theta[W(y,q_y)]h(y\cos(\theta) + q_y\sin(\theta) - s)ds\,d\theta$$



- Theorical reconstruction of Wigner function using analytic data taking marginal $0 < \theta < 2\pi$ distribution each $\frac{\pi}{60}$



- Experimental Wigner function using 16 marginal distribution measurements with angels $68° \leq \theta \leq 97°$

Conclusions and future work

- To reconstruct the Wigner function, it is necessary to consider marginal distributions between $0 < \theta < \pi$, as otherwise, the Wigner graph cannot be reconstructed effectively.
- Effective lenses formed using distances between them greater than 5cm generate transforms that do not match theoretical FFTs.
- Despite the presence of the two positive peaks in the experimental Wigner function, it is necessary to collect more data and attempt to reduce the noise in the images captured by the CCD to enhance the obtained results.

References

- Piñeros, P. (2022). *Emulating the Wigner function of a odd cat state by means of classical light fields* [Monographic]. Universidad de los Andes.
- *Martínez, A. (2019). Characterization of quantum states of light by means of homodyne detection and reconstruction of Wigner functions [Monographic]. Universidad de los Andes.*