On the Tomographic Reconstruction of the Wigner Distribution Function of an Odd Cat State Using Fractional Fourier Transform

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Theory

Wigner Distribution Function (WDF)

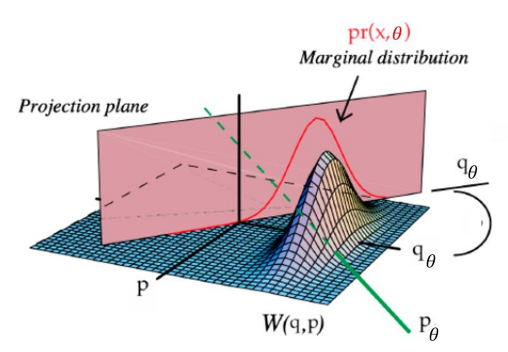
It's a quasi-probabilistic distribution function which allows us to analyze the wave function of a physical system in the phase space. It is called quasi-probabilistic "distribution" because it can reach negative values, while its projection on any axis in the phase space shows a probabilistic distribution.

$$W_u(y, q_y) = \int_{-\infty}^{\infty} u\left(y - \frac{y'}{2}\right) \overline{u}\left(y + \frac{y'}{2}\right) e^{-iq_y y'}\, dy'$$

Radon Transform (RT)

It projects any function embedded in the phase space on an axis rotated \theta from the q axis. This projection Is called a marginal distribution on that axis.

$$R_\theta(s) = \iint W(y, q_y)\, \delta(s - y\cos\theta - q_y \sin\theta)\, dy\, dq_y$$



Wigner function and its marginal distribution [5].

Fractional Fourier Transform (FrFT)

Is the concatenation of a Fourier Transforms in order to access different axes in the phase space than the usual p and q quadratures. Allowing us to access quadratures that are a combination of both, p and q.

Integral Definition of FrFT:

$$F_\alpha(u) := \int_{-\infty}^{\infty} f(x)\left(\frac{1 - i\cot\alpha}{2\pi}\right)^{1/2} \exp\left[\frac{i}{2}(x^2 + u^2)\cot\alpha - ixu\csc\alpha\right] dx$$

FrFT as a repeated application of the Fourier transform operator:

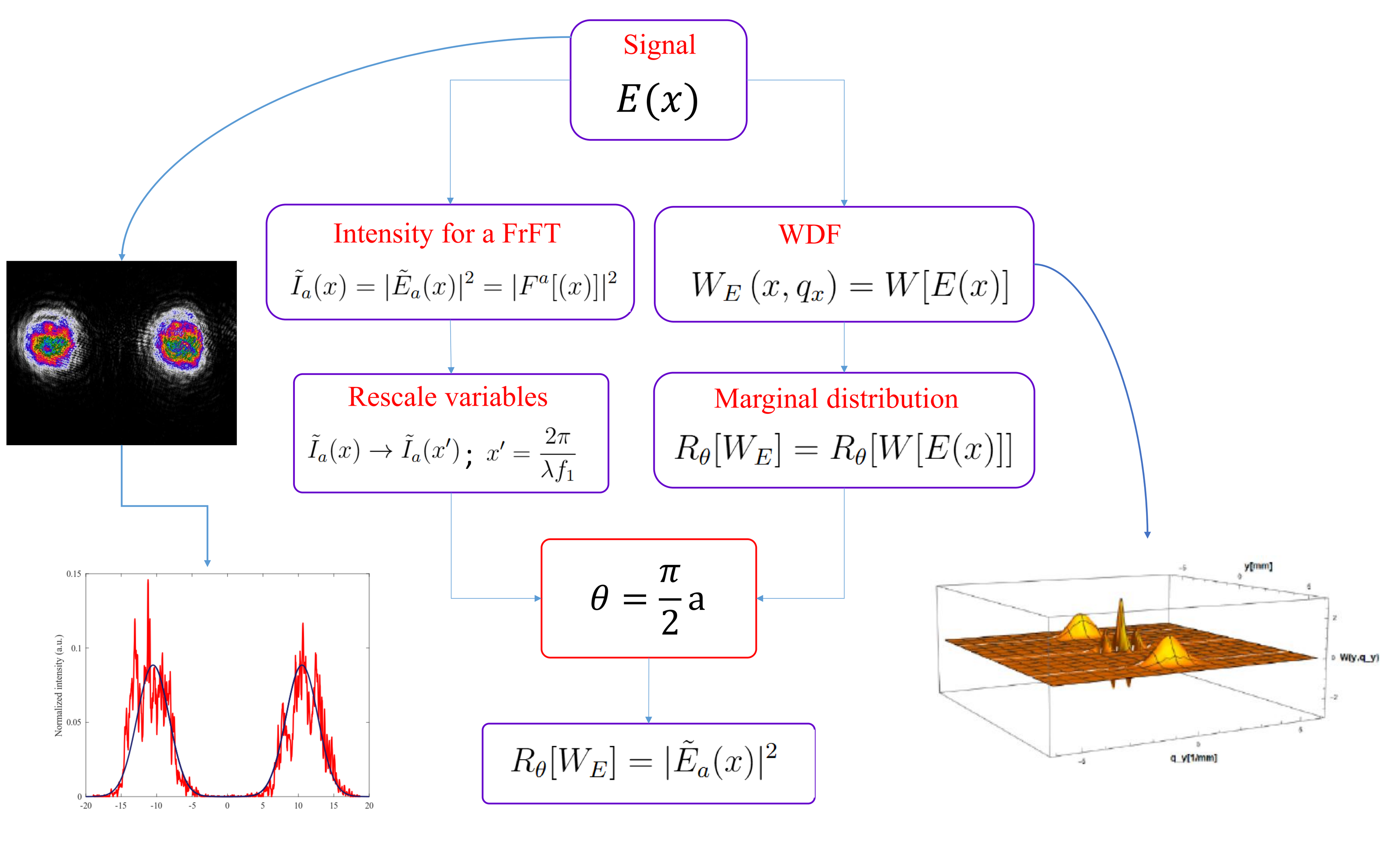
$$F^P\{f(x)\} := \Big(\underbrace{F \circ F \circ \cdots \circ F}_{P}\Big)\{f(x)\}$$

Additivity property:

$$F^{P_2}\{F^{P_1}\{u(x)\}\} = F^{(P_1+P_2)}\{u(x)\}$$

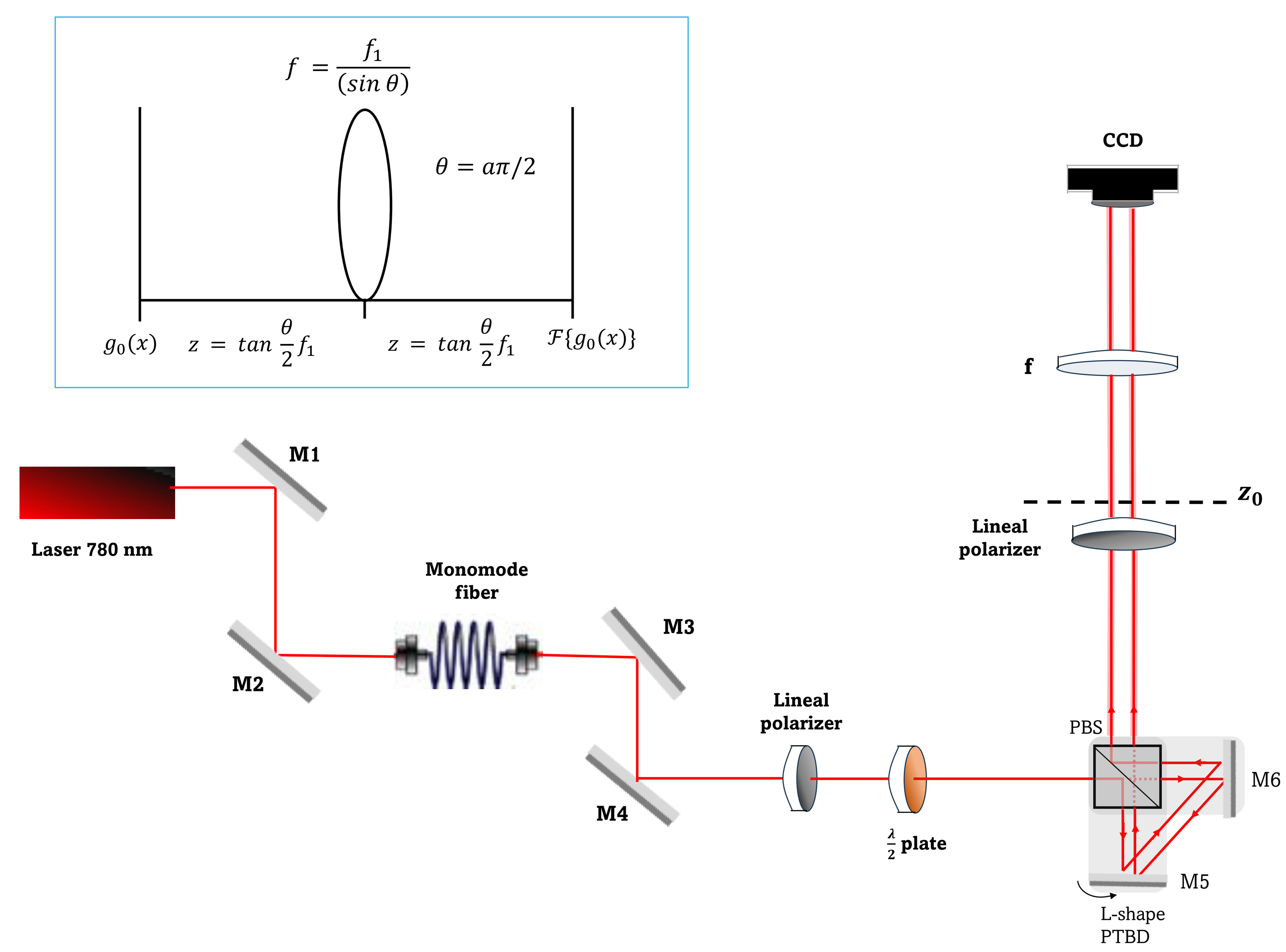
Relation between WFD and FrFT

The intensity we measure of an electromagnetic wave can be related with its WDF using the FrFT and RT as shown in the diagram below



Optical Set-up

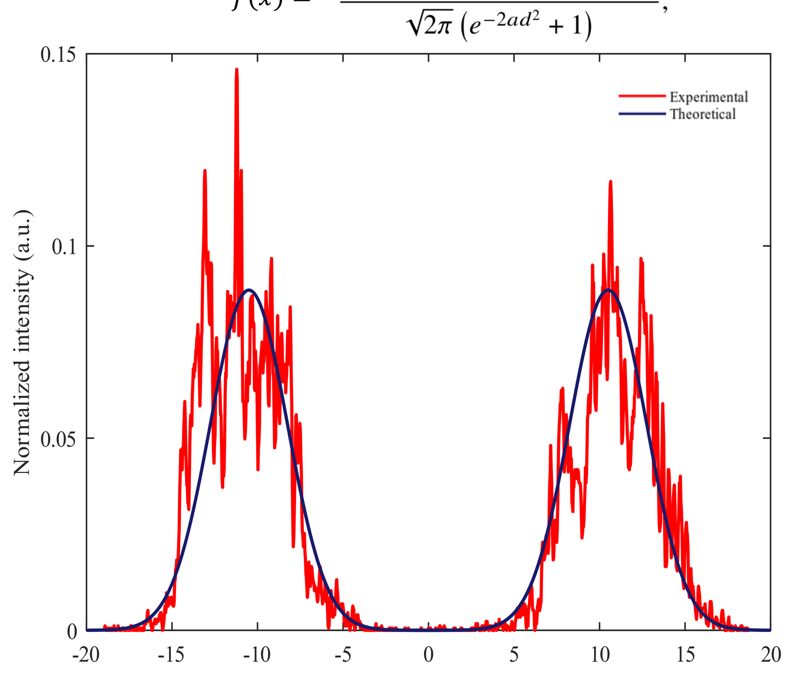
Propagation – lens – propagation scheme [1].



Results

Initial beam profile

$$f(x) = \frac{\sqrt{a}\left(e^{-a(x-d)^2} + e^{-a(d+x)^2}\right)^2}{\sqrt{2\pi}\left(e^{-2ad^2} + 1\right)},$$



Two Gaussian beams separated a distance 2d = 3.45 mm and a parameter a = 1.98 mm related to gaussian width as $1/\sqrt{2a}$.

Considerations

It is necessary to change the variables since experimentally $f(Lx)$ is measured, not $f(x)$. Where L is

$$L = \sqrt{\frac{\lambda f_1}{2\pi}}$$

So we are now left with

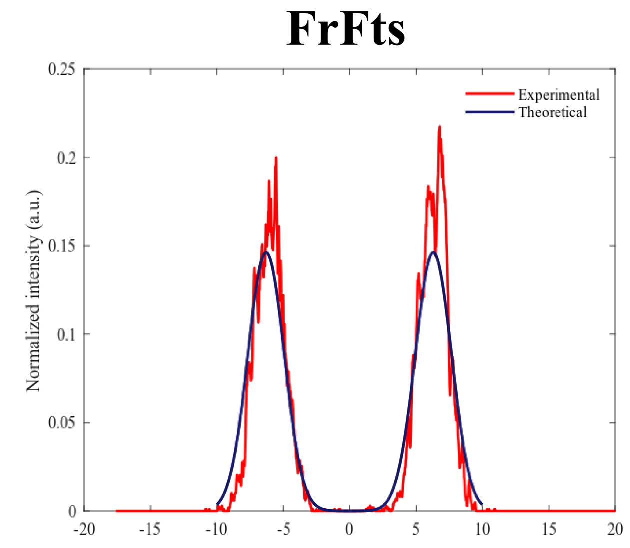
$$f'(x') = \frac{\sqrt{a'}\left(e^{-a'(x'-d')^2} + e^{-a'(d'+x')^2}\right)^2}{\sqrt{2\pi}\left(e^{-2a'd'^2} + 1\right)}$$

$$d' = d/L$$

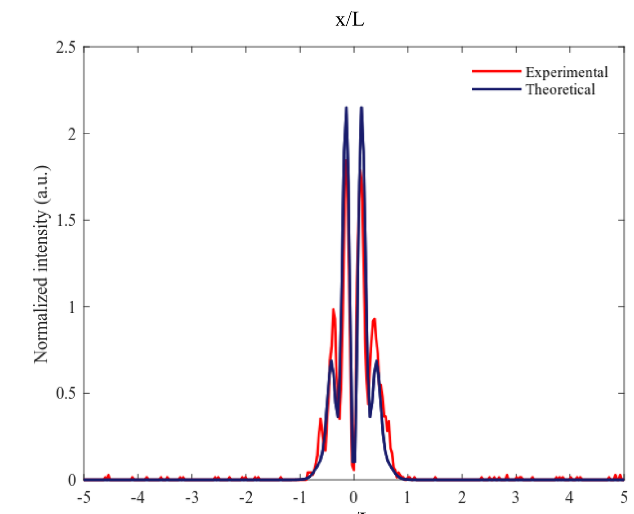
$$a' = aL^2$$

FrFts

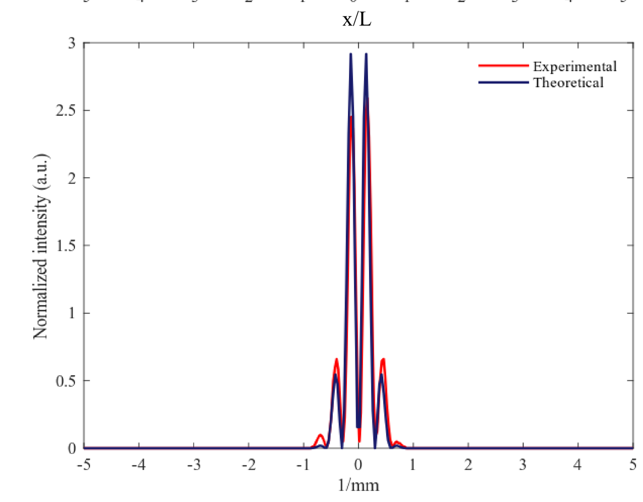
FrFT for θ = 53° = 0.927rad



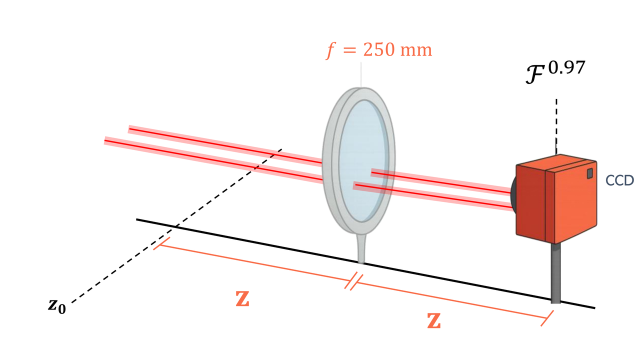
FrFT for θ = 88° = 1.55 rad

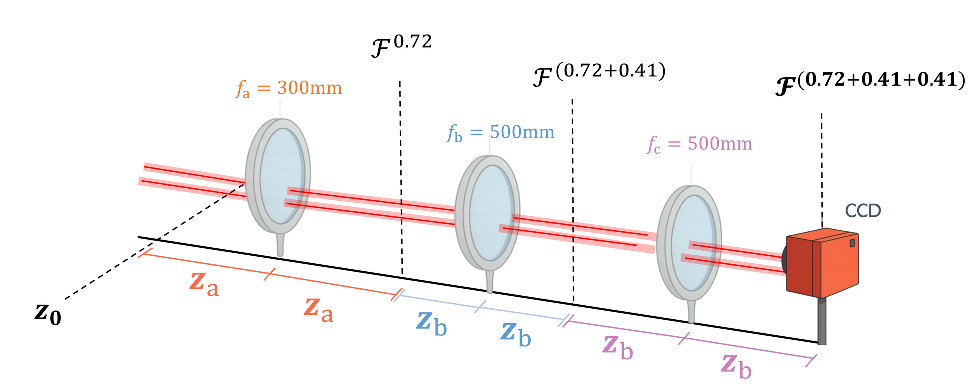


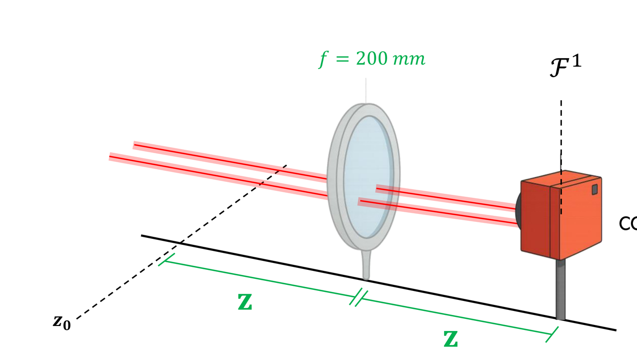
Full Fourier Transform $\theta = \frac{\pi}{2}$



Experimental set-ups used for the measurements







Conclusions and Future Work

- Perform other FrFT that allows us to access angles around 90° to collect marginal distributions with some interference structure.
- Use a Tomographic program to reconstruct the WDF with all the marginal distributions measured.
- Test different values for the width of the laser beams and the distance between them to optimize the measure technique.

References

[1] A. W. Lohmann, "Image rotation, Wigner rotation, and the fractional Fourier transform," JOSA A 10, 2181–2186 (1993). Publisher: Optica Publishing Group.
[2] A. W. Lohmann, Z. Zalevsky, R. G. Dorsch, and D. Mendlovic, "Experimental considerations and scaling property of the fractional Fourier transform," Opt. Commun. 146, 55–61 (1998).
[3]: Pedro Enrique PiñerosLourenco. Emulating the wigner function of an odd cat state by means of classical light fields. 2023.
[4]: SebastiánMenjura, Reconstruction of the Wigner function for a cat state using a classical Gaussian Beam. 2023
[5]: Martínez, A. 2020. Characterization of quantum states of light by means of homodyne detection and reconstruction of Wigner functions