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Curso: Proyecto Teórico

Understanding the Orbital Angular Momentum of Light with the aim of implementing high-dimensional quantum logic gates

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Introduction

Light has many degrees of freedom, including polarization, frequency, **Orbital Angular Momentum** (OAM), among others. OAM is a property of light beams with a helical phase structure, first demonstrated by Allen in 1992 while studying the behavior of **Laguerre-Gauss** beams [1,2]. Although research on these beams initially progressed slowly due to the demanding generation process, recent advancements in tools such as **Spatial Light Modulators** (SLM), which enable the manipulation of a beam's transverse phase profile, have facilitated the generation of OAM light beams. When treated quantum mechanically, OAM beams are quantized in integer multiples of $\hbar$ per photon, $\ell\hbar$. This integer value is, in principle, unbounded, meaning that the basis describing the OAM state of a photon can be very large [3]. This property makes OAM particularly useful for information storage and **high-dimensional quantum information processing**. For example, high-dimensional quantum computing is an emerging field that offers advantages over traditional qubit-based quantum computing, such as higher information density per physical entity and more efficient methods for developing quantum algorithms [4].

1. Light carrying Orbital Angular Momentum

A manner of which this kind of beams can appear is Laguerre-Gauss beams; these beams description appears when solving the **wave equation** for the electric field $\mathbf{\tilde{E}}(\mathbf{r},t) = \tilde{E}(\mathbf{r},t)\hat{\mathbf{e}}_i$, here $\hat{\mathbf{e}}_i$ is a polarization unit vector

$$\nabla^2 \tilde{E}(\mathbf{r},t) = \frac{1}{c^2}\frac{\partial^2 \tilde{E}(\mathbf{r},t)}{\partial t^2}$$

then, using a paraxial approximation one finds that

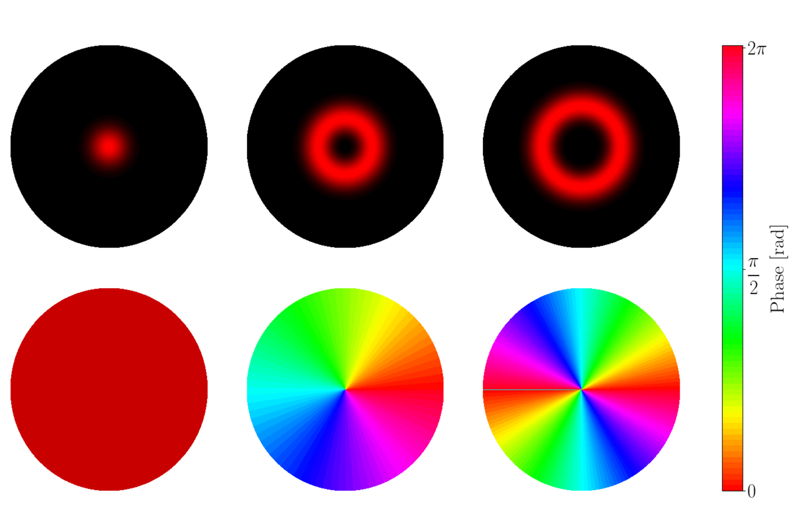
$$\nabla_\perp^2 \tilde{E}_0(\mathbf{r}) + 2ik\frac{\partial \tilde{E}_0(\mathbf{r})}{\partial z} = 0$$

where $\nabla_\perp^2$ is the transverse Laplacian, $\tilde{E}_0(\mathbf{r}) = \tilde{E}(\mathbf{r})e^{-ikz}$ and k is the wavenumber of the electromagnetic wave. When solving in cylindrical coordinates, the equation leads to a solution of the form

$$\tilde{E}_0(\mathbf{r}) = C_p^\ell \frac{w_0}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^\ell \times L_p^\ell\left(\frac{2r^2}{w^2(z)}\right) e^{-\frac{r^2}{w^2(z)} + ik\frac{r^2}{2R(z)} + i\ell\phi + i\psi(z)}.$$

Normalization constant
Topological charge OAM
Gouy's phase
Light beam's waist
Laguerre polynomials
Wave front radius

These are the so-called Laguerre-Gauss Modes of the electric field.



2. High-dimensional logic gates

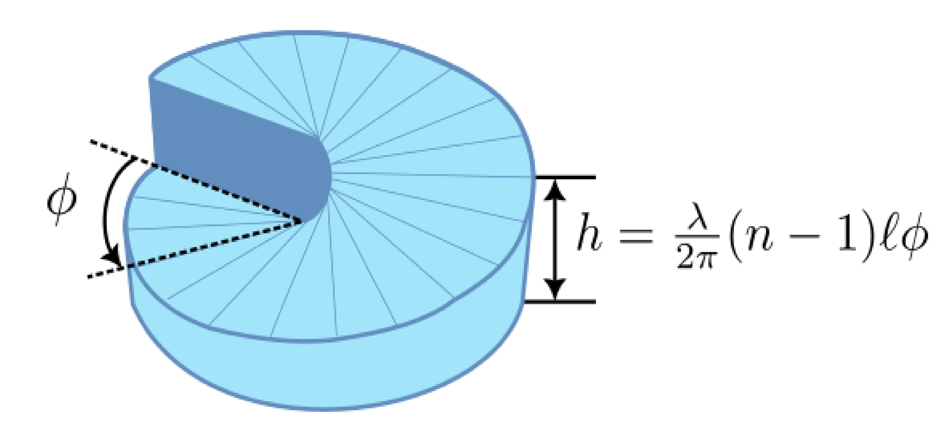
As in classical computing, quantum logic gates are the basis for any algorithm and information processing based on circuit logic. For quantum computing with qubits, transformations made by the Pauli matrices are those called X, Y and Z quantum gates. Thus, a generalization for the Pauli matrices in a d-dimensional space is needed. These are the d-dimensional Pauli operators, which can be written such as

$$\hat{X}_d = \sum_{\ell=0}^{d-1} |\ell \oplus 1\rangle\langle\ell| \quad \hat{Y}_d = \sum_{\ell=0}^{d-1} \omega^{\ell+1/2}|\ell \oplus 1\rangle\langle\ell| \quad \hat{Z}_d = \sum_{\ell=0}^{d-1} \omega^\ell|\ell\rangle\langle\ell|$$

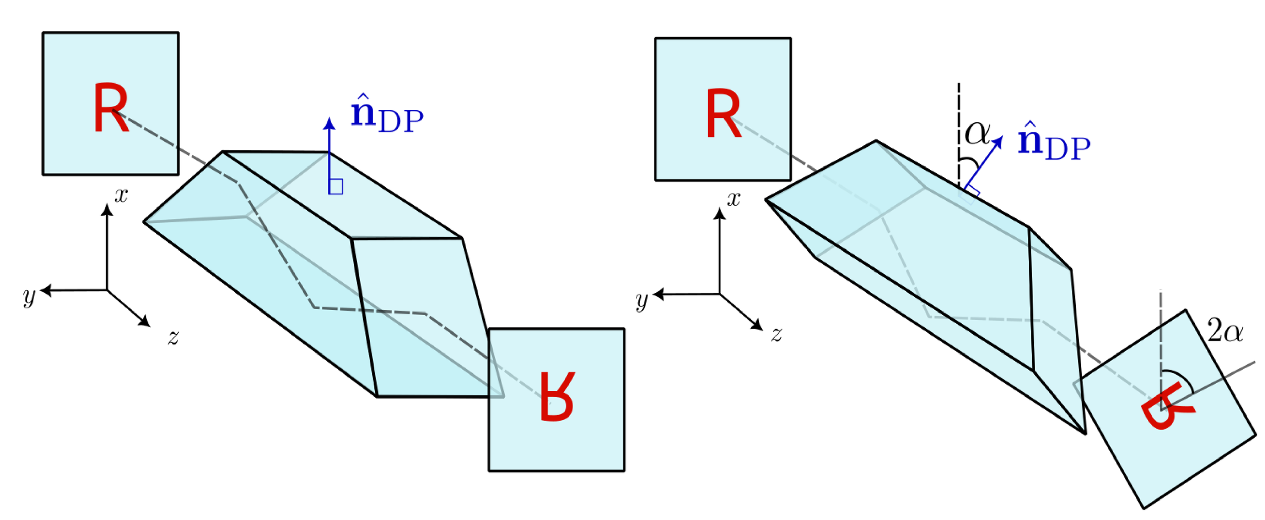
where $\oplus$ is the sum module d and $\omega = e^{\frac{2\pi i}{d}}$. These transformations are key elements for quantum information processing with high-dimensional quantum states [6].

3. Optical elements transforming OAM

- **Spiral Phase Plate (SPP):** Using a birefringent material with an azimuthally dependent thickness, a SPP adds an integer value to the topological charge ℓ to any incoming light beam



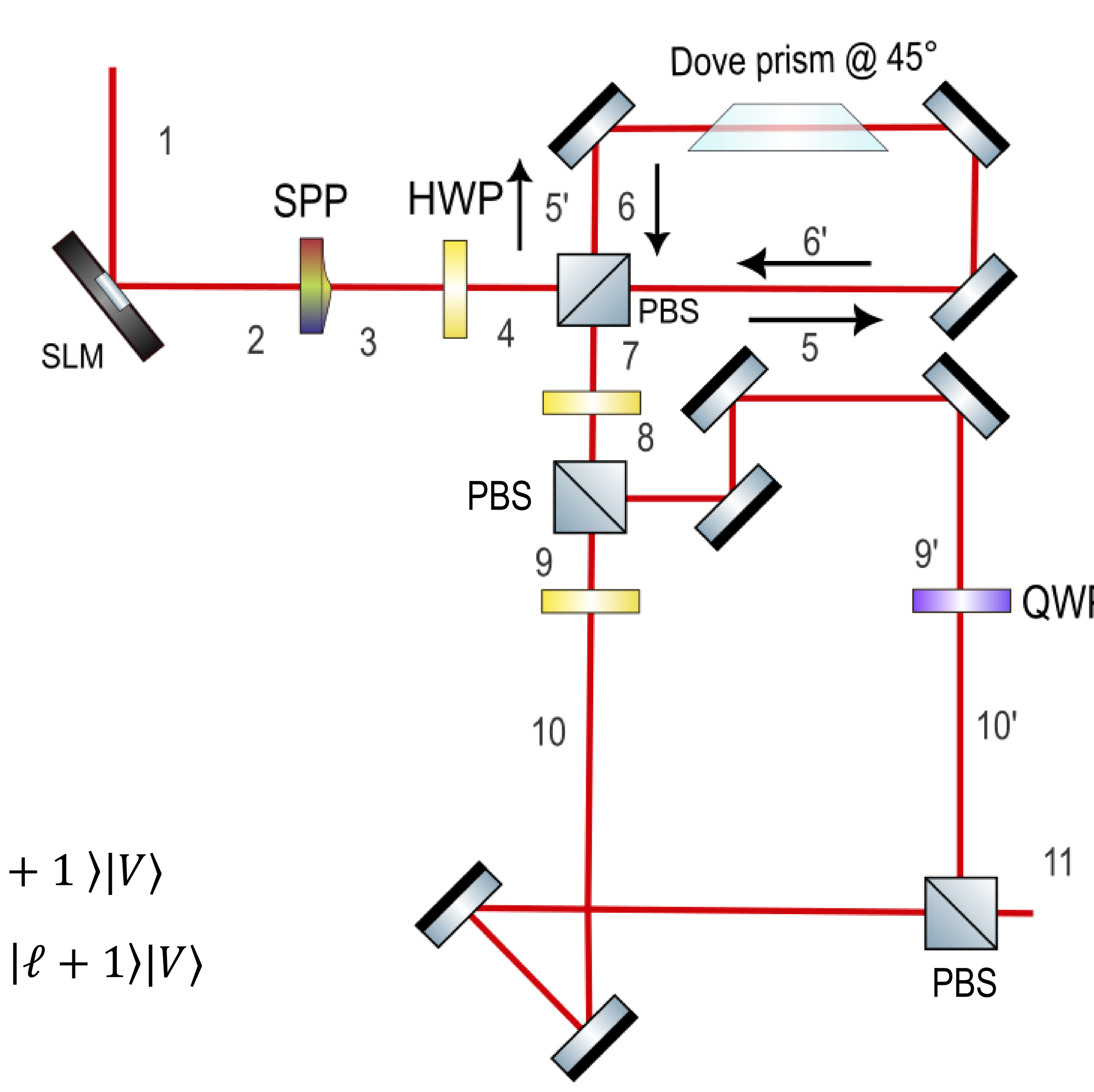
- **Dove Prism (DP):** This component acts on the beam adding a phase of 2α when rotated at an angle α. This component is very important as this is the exact same operation a $\hat{Z}_d$ is supposed to do.



4. Building a setup

In order to implement a X gate, the setup shown here is proposed as a way to apply a $\hat{X}_d$ transformation. This setup works as follows:

(1) $|\psi\rangle = |0\rangle|H\rangle$
(2) $|\psi\rangle = |\ell\rangle|H\rangle$
(3) $|\psi\rangle = |\ell+1\rangle|H\rangle$
(4) $|\psi\rangle = |\ell+1\rangle|D\rangle$
(5) $|\psi\rangle = |\ell+1\rangle|H\rangle$
(5') $|\psi\rangle = -|\ell+1\rangle|V\rangle$
(6) $|\psi\rangle = -e^{-\frac{i\pi}{2}(\ell+1)}|\ell+1\rangle|H\rangle$
(6') $|\psi\rangle = e^{\frac{i\pi}{2}(\ell+1)}|\ell+1\rangle|V\rangle$
(7) $|\psi\rangle = \frac{1}{\sqrt{2}}e^{-\frac{i\pi}{2}(\ell+1)}|\ell+1\rangle|H\rangle + \frac{1}{\sqrt{2}}e^{\frac{i\pi}{2}(\ell+1)}|\ell+1\rangle|V\rangle$
(8) $\cos\left(\frac{\pi}{2}(\ell+1)\right)|\ell+1\rangle|H\rangle + \sin\left(\frac{\pi}{2}(\ell+1)\right)|\ell+1\rangle|V\rangle$



In step (8) one can observe that even and odd modes of OAM are sorted by their associated polarization. After passing through the PBS2, odd modes won't experience any change on path (10), while even modes will undergo a sign flip due to an extra reflection on path (9'), so that in (11) reproduces a X gate transformation [7].

5. Conclusions and perspectives

- The theoretical behaviour and properties of Laguerre-Gauss Modes and high-dimensional logic gates were studied.
- Optical components that transform OAM and how they worked were studied with the aim of using them in an optical table.
- An optical setup which performs $\hat{X}_d$ is presented.
- The next step is to implement the setup shown here in the Quantum Optics Laboratory.

Bibliography

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